Exercises for Section 3.1

4. A system contains two components, A and B. The system will function so long as either A or B functions. The probability that A functions is 0.95, the probability that B functions is 0.90, and the probability that both function is 0.88. What is the probability that the system functions?

Solution:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= .95 + .90 - .88 = .97$$

6. Human blood may contain either or both of two antigens, A and B. Blood that contains only the A antigen is called type A, blood that contains only the B antigen is called type B, blood that contains both antigens is called type AB, and blood that contains neither antigen is called type O. At a certain blood bank, 35% of the blood donors have type A blood, 10% have type B, and 5% have type AB.
   a. What is the probability that a randomly chosen blood donor is type O?
   b. A recipient with type A blood may safely receive blood from a donor whose blood does not contain the B antigen. What is the probability that a randomly chosen blood donor may donate to a recipient with type A blood?

Solution:

a) $P(O) = 1 - P(A) - P(B) - P(AB)$
$= 1 - .35 - .1 - .05 = .50$

b) $P(B^C) = 1 - P(B) - P(AB) = 1 - .1 - .05$
$= .85$

Exercises for Section 3.2

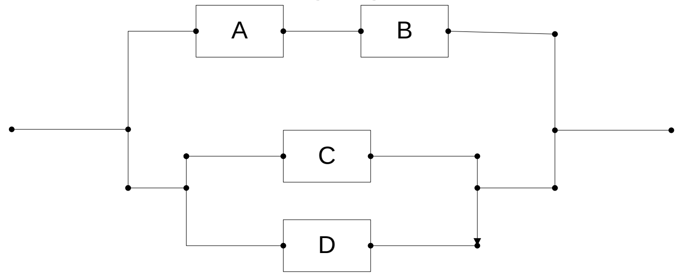
2. A drag racer has two parachutes, a main and a backup, that are designed to bring the vehicle to a stop after the end of a run. Suppose that the main chute deploys with probability 0.99 and that if the main fails to deploy, the backup deploys with probability 0.98.
   a. What is the probability that one of the two parachutes deploys?
   b. What is the probability that the backup parachute deploys?

Solution:

a) $P(M \cup B) = P(M) + P(B) - P(M \cap B)$
$= .99 + .98 - (.99)(.98) = .9998$

b) $P(B \cap M^C) = P(M^C) \times P(B|M^C)$
$= (1 - .99)(.98) = .0098$

6. A system consists of four components connected as shown in the following diagram:

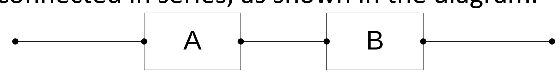


Assume A, B, C, and D function independently. If the probabilities that A, B, C, and D fail are 0.10, 0.05, 0.10, and 0.20, respectively, what is the probability that the system functions?

Solution:

$P(Subsys1) = P(A \cap B) = (1 - .10)(1 - .05) = .855$
$P(Subsys2) = P(C \cup D) = 1 - (.10)(.20) = .98$
$P(System) = P(Subsys1 \cup Subsys2)$
$= 1 - (1 - .855)(1 - .98) = .9971$

8. A system contains two components, A and B, connected in series, as shown in the diagram.



Assume A and B function independently. For the system to function, both components must function.
   a. If the probability that A fails is 0.05, and the probability that B fails is 0.03, find the probability that the system functions.
   b. If both A and B have probability p of failing, what must the value of p be so that the probability that the system functions is 0.90?
   c. If three components are connected in series, and each has probability p of failing, what must the value of p be so that the probability that the system functions is 0.90?

Solution:

a) $P(System) = P(A \cap B) = (.95)(.97) = .9215$

b) $P(System) = P(A \cap B) = (1 - p)(1 - p) = .90$
$\therefore p = .05132$

c) $P(System) = P(A \cap B \cap C) = (1 - p)(1 - p)(1 - p) = .90$
$\therefore p = .03451$

Exercises for Section 3.3

2. Computer chips often contain surface imperfections. For a certain type of computer chip, the probability mass function of the number of defects X is presented in the following table.

x	0	1	2	3	4
P(x)	0.4	0.3	0.15	0.10	0.05

   a. Find $P(X \leq 2)$.

b. Find P(X > 1).
c. Find μ_X
d. Find $\sigma_X{}^2$

Solution:

a) $P(X \le 2) = P(0) + P(1) + P(2) = .85$
b) $P(X > 1) = P(2) + P(3) + P(4) = .30$
c) $\mu_X = \sum xP(x) = 0(.4) + 1(.3) + 2(.15) + 3(.1) + 4(.05) = 1.1$
d) $\sigma_X{}^2 = \sum x^2 P(x) - \mu_X{}^2 = 0^2(.4) + 1^2(.3) + 2^2(.15) + 3^2(.1) + 4^2(.05) - 1.1^2 = 1.39$

6. After manufacture, computer disks are tested for errors. Let X be the number of errors detected on a randomly chosen disk. The following table presents values of the cumulative distribution function F(x) of X.

x	F(x)
0	0.41
1	0.72
2	0.83
3	0.95
4	1.00

a. What is the probability that two or fewer errors are detected?
b. What is the probability that more than three errors are detected?
c. What is the probability that exactly one error is detected?
d. What is the probability that no errors are detected?
e. What is the most probable number of errors to be detected?

Solution:

a) $P(X \le 2) = .83$
b) $P(X > 3) = 1 - P(X \le 3) = 1 - .95 = .05$
c) $P(X = 1) = P(X \le 1) - P(X \le 0) = .72 - .41 = .31$
d) $P(X = 0) = P(X \le 0) = .41$
e) 0

8 Elongation (in %) of steel plates treated with aluminum are random with probability density function

$$f(x) = \begin{cases} \frac{x}{250} & 20 < x < 30 \\ 0 & otherwise \end{cases}$$

a. What proportion of steel plates have elongations greater than 25%?
b. Find the mean elongation.
c. Find the variance of the elongations.
d. Find the standard deviation of the elongations.
e. Find the cumulative distribution function of the elongations.
f. A particular plate elongates 28%. What proportion of plates elongate more than this?

Solution:

a) $P(x > 25) = \int_{25}^{30} \frac{x}{250} dx = \frac{x^2}{500}\Big|_{25}^{30} = \frac{900}{500} - \frac{625}{500} = 0.55$
b) $\mu_x = \int_{20}^{30} x \frac{x}{250} dx = \frac{x^3}{750}\Big|_{20}^{30} = \frac{2700}{750} - \frac{800}{750} = 25.33$
c) $\sigma_x{}^2 = \int_{20}^{30} x^2 \frac{x}{250} dx - \mu_x{}^2 = \frac{x^4}{1000}\Big|_{20}^{30} - \mu_x{}^2 = \frac{810000}{1000} - \frac{160000}{1000} - 25.33^2 = 8.2222$
d) $\sigma_x = \sqrt{\sigma_x{}^2} = \sqrt{8.2222} = 2.8674$
e) $F(x) = \int_{-\infty}^{x} f(t)dt$

If $x < 20, F(x) = \int_{-\infty}^{x} 0dt = 0$

If $20 \le x < 30, F(x) = \int_{20}^{x} \frac{t}{250} dt = \frac{t^2}{500}\Big|_{20}^{x} = \frac{x^2}{500} - \frac{400}{500}$

If $x > 30, F(x) = \int_{-\infty}^{20} 0dt + \int_{20}^{30} \frac{t}{250} dt + \int_{30}^{x} 0dt = 1$

f) $P(x > 28) = 1 - F(28) = 1 - \left(\frac{28^2}{500} - \frac{400}{500}\right) = 1 - .768 = .232$

Exercises for Section 3.4

4. The force, in N, exerted by gravity on a mass of m kg is given by $F = 9.8m$. Objects of a certain type have mass whose mean is 2.3 kg with a standard deviation of 0.2 kg. Find the mean and standard deviation of F.

Solution:

$\mu_F = \mu_{9.8m} = 9.8\mu_m = 9.8(2.3) = 22.54N$
$\sigma_F = \sigma_{9.8m} = 9.8\sigma_m = 9.8(0.2) = 1.96N$

10. A gas station earns \$2.60 in revenue for each gallon of regular gas it sells, \$2.75 for each gallon of midgrade gas, and \$2.90 for each gallon of premium gas. Let X_1, X_2, and X_3 denote the numbers of gallons of regular, midgrade, and premium gasoline sold in a day. Assume that X_1, X_2, and X_3 have means $\mu_1 = 1500$, $\mu_2 = 500$, $\mu_3 = 300$, and standard deviations $\sigma_1 = 180$, $\sigma_2 = 90$, $\sigma_3 = 40$, respectively.

a. Find the mean daily revenue.
b. Assuming X_1, X_2, and X_3 to be independent, find the standard deviation of the daily revenue.

Solution:

a) $R = 2.60X_1 + 2.75X_2 + 2.90X_3$
$\mu_R = 2.60\mu_1 + 2.75\mu_2 + 2.90\mu_3 = 6145$
b) $\sigma_R = \sqrt{2.60^2\sigma_1{}^2 + 2.75^2\sigma_2{}^2 + 2.90^2\sigma_3{}^2} = 541.97$