Enhanced Fixed-Priority Scheduling with (m,k)-Firm Guarantee

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Abstract

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Keywords: ***Scheduling, Real-time system, Quality of Service, General Chinese Remainder Theorem, Overloading, Firm-deadline***

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1 Introduction

Much work has been conducted in scheduling analysis of hard real-time systems, where violating task deadlines must be avoided at all cost. However, in many real-time embedded systems, e.g., a video decoder, it is often acceptable to miss task deadlines occasionally. Several firm-deadline models have been proposed to study such systems, e.g., the imprecise computation model [4], the "skip-over" model [12], and the (m,k)-firm guarantee model [9]. In the (m,k) model ($0 < m \leq k$), system dynamic failure occurs if fewer than m out of any k consecutive jobs of some task meet their deadlines. If $m = k$, the system becomes a hard-deadline system. For the special case of $m = k - 1$, the (m,k) model reduces to the "skip-over" model [12]. The (m,k) model can be readily incorporated into system Quality of Service (QoS) requirements, and is applicable to many real-time systems such as those in multimedia and automotive control. In this paper, we use the (m,k) model to study the scheduling problem of overloaded systems.

Some approaches [1, 2, 3, 5, 7, 9, 12, 19] apply dynamic scheduling techniques to handle overloaded real-time systems. However, in many applications, fixed-priority scheduling algorithms are usually more attractive than dynamic-priority ones because (i) it incurs lower overhead; (ii) the implementation is relatively simple; (iii) it gives a designer control over task priorities. In this paper, we focus on applying fixed-priority scheduling to deal with overloaded systems. A few papers have been published that study the (m,k) model under fixed-priority scheduling. In [12], the "skip-over" model is used and the task set schedulability is analyzed in that context. However, the results cannot be readily applied to the (m,k) model. In [16], Ramanathan proposed a scheduling technique for the general (m,k) model. The beauty of the technique is that it uses a very simple algorithm to partition the jobs of each task into two sets: mandatory and optional. All mandatory jobs are scheduled according to their fixed-priorities, while optional jobs are assigned the lowest priority. It follows that if all mandatory jobs meet their deadlines, no dynamic failure will happen.

Though the technique proposed in [16] is simple and elegant, it does have some potential problems. First, the *first* job of every task is always designated as mandatory, which forces the worst case response time of every task to be that of the first job. Secondly, the job partition algorithm implicitly distributes the mandatory jobs evenly among k consecutive instances of a task. Such even distribution may not be advantageous in certain situations. Furthermore, the partition algorithm depends solely on the ratio of m over k of each task. That is, regardless of task periods and execution times, the mandatory jobs of two tasks having the same m over k ratio are always distributed in the same way among the k consecutive jobs. In the following, we provide some examples to illustrate the consequence of the above problems. In summary, all the above problems can significantly impact task set schedulability, which may then lead to overly pessimistic designs.

We believe that judicious selection of mandatory *v.s.* optional jobs plays a critical role in scheduling systems with (m,k)-firm constraints. In this paper, we first prove that the problem of finding the *optimal* partition between mandatory and optional jobs for each task is NP-hard in the strong sense. Then, we present a heuristic algorithm to modify the partitions given in [16]. Through analyzing the effects of preemption and blocking on lower priority mandatory jobs by higher priority ones, we design an algorithm to carefully select mandatory jobs and reduce such effects. Our experimental results show that our algorithm produces significantly better partitions than the ones proposed in [16] in terms of system schedulability. We also formally show that our solutions form a super set of that obtained by [16], in the sense that any task set with (m,k) constraints schedulable by [16] is always schedulable by using our algorithm.

The schedulability of (m,k) systems can be further improved if one can tolerate spending some more time on finding better mandatory/optional partitions off-line. In this regards, we believe that a probabilistic optimization algorithm (e.g., genetic algorithms or simulated annealing) can be very effective. One challenge in applying such algorithms is to formulate an appropriate objective function. We propose a metric that can be used as an objective function and demonstrate its effectiveness by implementing a genetic algorithm based on this metric. The experimental results are extremely encouraging.

Another difficulty is to determine the schedulability of tasks with (m,k) constraints after mandatory jobs are selected, which we prove to be NP-hard. One way to solve this problem is to perform the exact analysis for a large number of possible cases as suggested in [1, 18], which is computationally intractable for large task sets. We present a sufficient condition which can be used to determine in polynomial time if a given set of mandatory jobs is schedulable. The condition was derived based on an extension to the algorithm presented in [10].

The paper is organized as follows. In Section 2, we define our problem and analyze some related work. In Section 3, we prove several theorems to demonstrate some characteristics of the (m,k)-firm guarantee problem and then introduce an important concept, *execution interference*, to capture the preemption and blocking effects among tasks. Section 4 contains a detailed discussion of our partitioning algorithms and approach to checking schedulability of task set with (m,k) constraints. Experimental results are given in Section 5. Finally, we summarize our work in Section 6.

2 Preliminaries and Motivation

Consider a system with n independent periodic tasks, $\mathcal{T} = \{\tau_1, \tau_2, \cdots, \tau_n\}$, arranged in the decreasing order of their priorities. Each instance of a task is called a *job*. The jth job of τ_i is denoted as τ_{ij}. The following timing parameters are defined for task τ_i:

- O_i: the release time of the first job of τ_i, referred to as *initial time*.
- T_i: the interval between two consecutive job release times of τ_i, referred to as *period*.
- D_i: the maximum time allowed from the release to the completion of τ_i's job, referred to as *deadline*.
- C_i: the maximum time needed to complete τ_i without any interruption, referred to as *execution time*.
- m_i and k_i: the (m,k) constraint for τ_i, which mandates that at least m out of k consecutive jobs of τ_i must be completed prior to or on their deadlines to avoid any dynamic failure.

When scheduling a task set with (m,k) constraints according to a fixed-priority assignment, one critical step is to determine for each task whether its execution is mandatory or optional. This may be envisioned as each job being associated with a binary variable π. If $\pi = 1$, the corresponding job is mandatory. Otherwise, it is optional. The collection of all these binary variables forms a binary string, which we refer to as the *mandatory job pattern*. It can be readily observed that the selection of the mandatory job pattern for each task may greatly impact the schedulability of the task set. To ease the computational requirement in searching for the mandatory job patterns that can satisfy the (m,k) constraints, we first introduce the following definition.

Definition 1 *The* ***(m,k)-pattern*** *of task τ_i, denoted by Π_i, is a binary string $\Pi_i = \{\pi_{i1}\pi_{i2}...\pi_{ik_i}\}$ which satisfies the following: (i) τ_{ij} is a mandatory job if $\pi_{ij} = 1$ and optional if $\pi_{ij} = 0$, and (ii) $\sum_{j=1}^{k_i} \pi_{ij} = m_i$.*

By repeating the (m,k)-pattern Π_i, we get a mandatory job pattern for τ_i. It is not difficult to see that the (m,k) constraints for τ_i can be satisfied if the mandatory jobs of τ_i are selected accordingly. Note that the length of the (m,k)-pattern for task τ_i is k_i. Although we could increase the length of the pattern to longer than k_i to improve the flexibility of selecting mandatory job patterns, this would increase the complexity of scheduling analysis and complicates system implementation at the same time. Consider, for example, the length of the mandatory job pattern is set to $2k_i$. Since requiring $\sum_{j=1}^{2k_i} \pi_{ij} = 2m_i$ does not necessarily guarantee satisfying the (m,k) constraint, one would need to check $2k_i$ windows of size k_i each (wrap around the pattern if necessary) in order to guarantee that the (m,k) constraint is not violated. Thus, longer mandatory job patterns would significantly increase the computation cost.

With the definition of (m,k)-pattern, we formulate the fixed-priority (m,k) scheduling problem as follows.

Definition 2 *Given a periodic task set $\mathcal{T}$, let the mandatory jobs defined by a set of (m,k)-patterns be assigned fixed priorities and the optional jobs have the lowest priority. Find the optimal (m,k)-pattern Π_i for each $\tau_i \in \mathcal{T}$ such that no other (m,k)-patterns can satisfy the (m,k) constraints if the optimal pattern cannot satisfy the (m,k) constraints.*

Solving the above problem consists of two challenges:

- given a task set with (m,k) constraints, how to determine if one set of (m,k)-patterns are better or easier to be scheduled than another;
- given a set of (m,k)-patterns, how to predict if the corresponding mandatory jobs are all schedulable.

In [12], the authors consider the "skip-over" model, a special case of the above fixed-priority (m,k) scheduling problem where $m = k - 1$. They prove that determining whether a set of periodic, occasionally skippable tasks is schedulable is NP-hard in the weak sense. We will extend their proof and show that the problem of finding the optimal (m,k)-patterns is NP-hard in the strong sense. When applying the rate-monotonic scheduling algorithm in the "skip-over" model, the authors in [12] implicitly adopt the so-called *deeply-red* task set to be the mandatory job set. This corresponds to the following (m,k)-pattern:

$$\pi_{ij} = \begin{cases} 1 & 1 \leq j < k_i - 1 \\ 0 & j = k_i \end{cases} \tag{1}$$

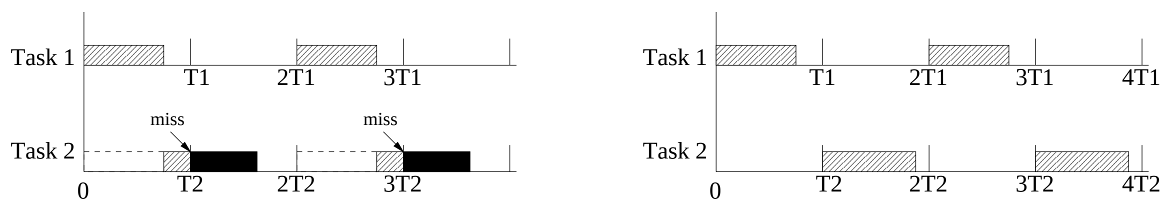
For the above (m,k)-pattern, a sufficient and necessary condition is presented in [12] to determine the schedulability. It is claimed in [12] that the worst case occurs in the deeply-red task set in the "skip-over" model. However, no further work is done on choosing different (m,k)-patterns to improve the schedulability of a task set.

In [16], the general (m,k) model is used and an algorithm is proposed for determining the (m,k)-patterns for a given task set, which leads to the following (m,k)-pattern:

$$\pi_{ij} = \begin{cases} 1 & \text{if } j = \lfloor \lceil \frac{(j-1)\times m_i}{k_i} \rceil \times \frac{k_i}{m_i} \rfloor + 1 \\ 0 & \text{otherwise} \qquad\qquad\qquad j = 1, 2, \cdots, k_i \end{cases} \tag{2}$$

For the (m,k)-patterns above, one can see that the (m,k)-pattern for a task is fixed once its (m,k) constraint is defined, and the first job of every task is always labeled to be mandatory. Moreover, it is proved in [16] that the algorithm results in the most mandatory jobs from $[0, t]$ compared with those in any other interval of the same length t. One attractive consequence of the approach in [16] is that the schedulability analysis can be conducted by simply extending that proposed in [13], since the first job of each task always has the worst case response time. However, this advantage becomes less desirable in terms of meeting (m,k) constraints.

Consider the example in Figure 1. Here, the task set contains two tasks with the same periods and the same (m,k)-firm constraint, i.e., (1,2). It is shown in Figure 1(a) that the mandatory jobs cannot be scheduled if the (m,k)-patterns are assigned according to (2), while some different (m,k)-patterns can satisfy the (m,k) constraints (see Figure 1(b)). In addition to forcing the worst case



(a) Using 10 as the (m,k)-patterns for both Task 1 and Task 2 results in an infeasible schedule

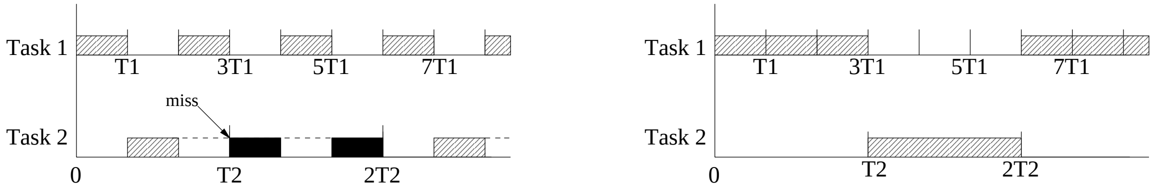
(b) Using 10 as the (m,k)-pattern for Task 1 and 01 for Task 2 results in a feasible schedule

Figure 1: Different (m,k)-patterns for the same task set lead to different scheduling results.

response time of every task to be that of the first job, the technique in [16] implicitly distributes the mandatory jobs evenly among k_i consecutive jobs of τ_i. Such even distribution may not be desirable in certain situations as seen in the example given in Figure 2, where the (m,k) constraint of τ_1 is $(3, 6)$ and that of τ_2 is $(1, 2)$.

In the following, we present our contributions on solving the (m,k) scheduling problem. We first introduce the term *work load* similar to the one used in [13]. It will be used extensively in the rest of the paper.

Definition 3 *Let t and $t + t'$ be two time instants in some τ_i-*busy period *[11]. The **work load** of τ_i*



(a) Using 101010 as the (m,k)-pattern for Task 1 and 10 for Task 2 results in an infeasible schedule

(b) Using 111000 as the (m,k)-pattern for Task 1 and 01 for Task 2 results in a feasible schedule

Figure 2: Evenly distributed mandatory jobs may not always improve the schedulability.

in [$t, t+t'$], denoted by $W_i(t, t+t')$, is defined as

$$W_i(t, t+t') = \sum_{j \leq i} l_{ij} \times C_j, \tag{3}$$

where l_{ij} is the number of mandatory jobs of τ_j ($j \leq i$) with their release times within [$t, t+t'$).

3 Observations on the (m,k) Scheduling Problem

In this section, we first present several observations related to the complexity issues of the (m,k) scheduling problem. Then, we discuss an important concept for estimating preemption and blocking effects among tasks with (m,k) constraints.

3.1 Complexity issues

We first show that selecting the "optimal" (m,k)-pattern for each task can be very "difficult".

Theorem 1 *Given a task set $\mathcal{T}$ the problem of deciding if there exists an (m,k)-pattern for each task in $\mathcal{T}$ such that $\mathcal{T}$ is schedulable is NP-hard in the strong sense.*

Proof: We prove the theorem by reducing the *3-Partition problem* to our scheduling problem. The *3-Partition problem* is: given a set $A = \{a_1, a_2, \cdots, a_{3m}\}$ of $3m$ positive integers and a positive integer B such that $\frac{1}{4}B < a_i < \frac{1}{2}B$ and $\sum_{i=1}^{3m} a_i = mB$, can A be partitioned into m disjoined sets, $A_1, A_2, ..., A_m$, such that $\sum_{a_i \in A_j} a_i = B$ for each $1 \leq j \leq m$? The *3-Partition problem* is proved to be NP-hard in the strong sense [6].

Given a *3-Partition problem*, we construct a task set $\mathcal{T} = \{\tau_1, \tau_2, \cdots, \tau_{3m}\}$ such that $O_i = 0, C_i = a_i, D_i = T_i = B, m_i = 1, k_i = m$. Assume we have found an (m,k)-pattern for each τ_i

such that $\mathcal{T}$ is schedulable. Then, after clustering tasks with the same (m,k)-pattern to form $\mathcal{T}_i'$ and let the corresponding a_j form A_i, we have

$$\mathcal{T}_i' \text{ is schedulable } \Longleftrightarrow \sum_{a_j \in A_i} a_j = B, i = 1, \cdots, m$$

Since the above reduction is linear, we prove the theorem. □

Another challenge in solving the (m,k) scheduling problem is to decide if the mandatory jobs given by a set of (m,k)-patterns are schedulable. Unfortunately, the problem is also NP-hard.

Theorem 2 *Given a task set $\mathcal{T}$ and an (m,k)-pattern for each task in $\mathcal{T}$, the problem of determining whether $\mathcal{T}$ is schedulable is NP-hard.*

Proof: Leung and Merrill have shown that checking the feasibility of a periodic task sets with arbitrary initial times is NP-hard [14]. For any task set $\mathcal{T}$ considered in [14], we can always construct a new task set $\mathcal{T}'$ with (m,k) constraints such that $m_i = k_i$ for all $0 \leq i \leq n$. The theorem holds because it has been proved in[14] that deciding whether $\mathcal{T}$ is schedulable or not is NP-hard. □

In Section 2, we reviewed the deeply-red task set used by the "skip-over" model in [12] and showed its (m,k)-pattern in (1). Here, we extend the deeply-red task set definition to the general (m,k)-firm guarantee model.

Definition 4 *Given a task set $\mathcal{T}$ with (m,k) constraints, the* ***deeply-red (m,k)-pattern*** *for task τ_i, $\Pi_i^r = \{\pi_{i1}^r \pi_{i2}^r ... \pi_{ik_i}^r\}$, satisfies*

$$\pi_{ij}^r = \begin{cases} 1 & 1 \leq j \leq m_i \\ \\ 0 & m_i < j \leq k_i \end{cases}$$

For the deeply-red (m,k)-pattern, we have the following observation.

Theorem 3 *For task set $\mathcal{T}$ with $O_i = 0, 1 \leq i \leq n$, if the mandatory jobs defined by the deeply-red (m,k)-patterns are schedulable, the mandatory jobs derived from any other (m,k)-patterns are also schedulable.*

Proof: Given the mandatory jobs according to the deeply-red (m,k)-patterns, for the first job of $\tau_i \in \mathcal{T}$, its work load in $[0, t]$ is,

$$W_i(0,t) = \sum_{j \leq i} C_j \times l_j,$$

where l_j is the number of mandatory jobs of τ_j in $[0, t]$. If τ_i is schedulable, there exists a time instant t_0 such that

$$W_i(0, t_0) = t_0 \leq D_i.$$

Suppose that a job of τ_i, τ_{iq}, has the worst case response time and is released in some τ_i-busy period [11]. A job from a higher priority task can interfere with the execution of τ_{iq} if it is released prior to r_{iq} but has not been completed by r_{iq} or it is released in the τ_i-busy period after r_{iq} (see

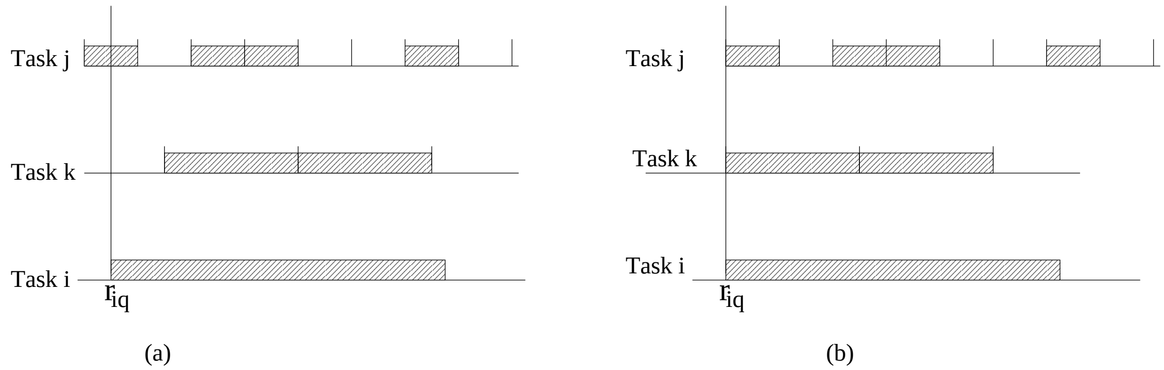


Figure 3: Shifting periods of higher priority tasks (τ_j and τ_k) to create a worse case for τ_i.

Figure 3(a)). If we shift the execution of every higher priority task such that its first job that interferes with τ_{iq} is released exactly at r_{iq}, the resultant job pattern will make τ_{iq} more difficult to be scheduled. Consequently, if τ_{iq} in Figure 3(b) is schedulable, so is τ_{iq} in 3(a), and hence τ_i is schedulable. In Figure 3(b), the work load of τ_i in $[r_{iq}, r_{iq} + t_0]$ is

$$W_i'(r_{iq}, r_{iq} + t_0) = \sum_{j \leq i} l_j' \times C_j,$$

where l_j' is the number of mandatory jobs of $\tau_j (j \leq i)$ in $[r_{iq}, r_{iq} + t_0]$.

Since l_j is the maximum possible number of mandatory jobs of τ_j $(j < i)$ within any interval with length t_0, i.e., $l_j' \leq l_j$, we have,

$$W_i'(r_{iq}, r_{iq} + t_0) \leq W_i(0, t_0) = t_0 \leq D_i.$$

and task τ_i must be schedulable. $\square$

3.2 Execution interference among tasks

As mentioning in previous sections, determining the schedulability of a task set with (m,k) constraints is a challenging problem, since exact timing analysis for a large number of possible cases is very time consuming and in fact intractable for large task sets. To reduce the computational cost, we propose an effective way to help characterize and quantify the preemption and blocking effects on lower priority mandatory jobs by higher priority ones.

Given two tasks τ_h and τ_i ($h < i$), we say that a τ_h's job *interferes* a τ_i's job if the execution time interval of the τ_h's job either partially or entirely overlaps with the period of the τ_i's job. We

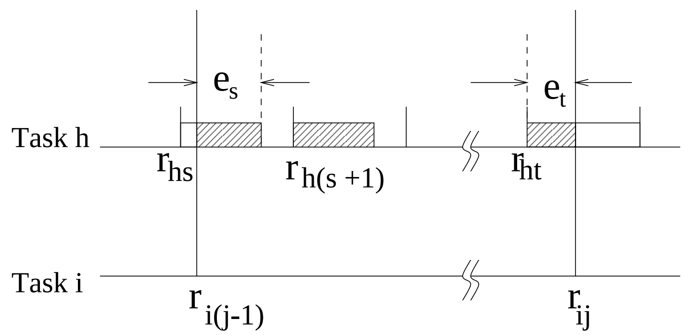


Figure 4: Execution interference of τ_h with τ_{ij}, where r_{pq} is the release time of job τ_{pq}.

use the term *execution interference* of τ_h with job τ_{ij} to capture the amount of *potential* preemption and/or blocking effect caused by τ_h during $[(j-1)T_i+O_i, jT_i+O_i]$. In Figure 4, τ_{hs},$\tau_{h(s+1)}$, and τ_{ht} all interfere with τ_{ij}, and the execution interference of τ_h with τ_{ij} is shown by the shaded regions. Formally, we define execution interference as follows.

Definition 5 *Given two tasks τ_h and τ_i ($h < i$) and the (m,k)-pattern for each task, the **execution interference** of τ_h with job τ_{ij}, denoted by F^h_{ij}, equals the total portions of the execution times of all τ_h's mandatory jobs that fall inside* $[(j-1)T_i + O_i, jT_i + O_i]$.

(Note that in Figure 4, e_s and e_t become zero if the corresponding jobs are not mandatory). Mathematically, F^h_{ij} can be calculated as follows,

$$F^h_{ij} = e_s + l^h_{ij} \times C_i + e_t, \tag{4}$$

where l^h_{ij} is the number of mandatory jobs of τ_h that fall entirely in the interval $[(j-1)T_i+O_i, jT_i+O_i]$, $e_s = \pi_{hs} \min\{C_h + r_{hs} - r_{i(j-1)}, 0\}$, and $e_t = \pi_{ht} \min\{C_h, r_{ij} - r_{ht}\}$.

Each mandatory job of τ_i may suffer different amount of interference by τ_h, and the job of τ_i that suffers the most execution interference from higher priority tasks tends to dominate the schedulability of τ_i. We refer to this maximum execution interference as the execution interference of task τ_h with task τ_i, and denote it by $\mathcal{F}_i^h$, i.e.,

$$\mathcal{F}_i^h = \max_j \{F_{ij}^h\}, j = 1, 2, \cdots.$$

Since there exists an infinite number of mandatory jobs for task τ_i, it might seem daunting to determine $\mathcal{F}_i^h$. To tackle this problem, we borrow an existing theorem, **Generalized Chinese Remainder Theorem** (GCRT)[15], which is restated below.

Theorem 4 (GCRT) *Let $v_1, v_2, \cdots, v_r$ be positive integers, v be the least common multiple of $v_1, v_2, \cdots, v_r$, and $a, u_1, \cdots, u_r$ be any integers. There exists exactly one integer u which satisfies $a \leq u < a+v$ and $u = u_j \pmod{v_j}$ for all $1 \leq j \leq r$ if and only if $u_i = u_j \pmod{\gcd(v_i, v_j)}$ for all $1 \leq i < j \leq r$, where $\gcd(x, y)$ denotes the greatest common divisor (GCD) of x and y.*

(Note that $a = b \pmod{c}$ is equivalent to $a \bmod c = b \bmod c$.) Based on GCRT, we proof two lemmas to be used for analyzing the execution interference between tasks. For generality, we use “events” rather than “tasks” in the lemmas.

Lemma 1 *Given two periodic events E_1 and E_2 with period T_1 and T_2, respectively, let the initial times of the two events be the same, i.e., $O_1 = O_2$. Denote the release time of any instance of E_1 (resp., E_2) by r_1 (resp., r_2). Then, $r_1 - r_2 = q * \gcd(T_1, T_2)$, $q \in Z$ (Z is the set of integers).*

Proof: Consider the case where $r_{1i} > r_{2j}$. (The other case can be proved in a similar manner.) Since $r_1 = aT_1$ and $r_2 = bT_2$ ($a, b \geq 0, \in Z$), we have

$$r_{1i} = z_1 * T_1,$$

and

$$r_2 = k_2 * T_2,$$

where k_1, $k_2 > 0$ and $k_1, k_2 \in Z$. Because

$$r_1 \bmod T_1 = r_2 \bmod T_2 = 0.$$

Then, by applying GCRT, we have

$$r_2 = r_1 \pmod{\gcd(T_1, T_2)}.$$

Hence,

$$|r_2 - r_1| = q * \gcd(T_1, T_2), \qquad q \geq 0, \text{ and } q \in Z.$$

$\square$

Lemma 1 states that the interval between the release times of any two instances of two periodic events always equals the integer multiple of the GCD of their periods, if these two periodic events have the same initial time. Similarly, for periodic events having different initial times, we have the following lemma.

Lemma 2 *Suppose that two periodic events E_1 and E_2 have periods T_1 and T_2, and different initial times O_1 and O_2, respectively. Denote the release time of any instance of E_1 (resp., E_2) by r_1 (resp., r_2). Then, $r_1 - r_2 = p * g + (O_1 - O_2) \bmod g$, where $g = \gcd(T_1, T_2)$, $p \in Z$. Furthermore, if $|r_1 - r_2|$ is the* minimum distance *between the release times of any E_1's instance and any E_2's instance, then $|r_1 - r_2| \leq \frac{g}{2}$.*

Proof: According to Lemma 1, if E_1 and E_2 had the same initial time, we would have

$$r_2 = r_1 + a * g, a \in Z.$$

For different initial times, O_1 and O_2, it follows that r_{1i} and r_{2j} satisfy one of the following:

$$r_2 - r_1 = a * g + (O_2 - O_1) = b * g + (O_2 - O_1) \bmod g,$$

and

$$r_1 - r_2 = a * g + (O_1 - O_2) = c * g + (O_1 - O_2) \bmod g,$$

where $b, c \in Z$. Since

$$\min |r_1 - r_2| = \min\{(O_1 - O_2) \bmod g, (O_2 - O_1) \bmod g\},$$

we conclude

$$\min |r_1 - r_2| \leq \frac{g}{2}$$

$\square$

Observe that τ_i's mandatory jobs corresponding to bit $\pi_{ij} = 1$ can be viewed as a periodic event E_i with period $k_i T_i$ and initial time $O_i + (j-1)T_i$. Furthermore, the mandatory jobs of τ_h

can also be viewed as a periodic event E_h with period k_hT_h and initial time O_h. Let the release time of an instance of E_i (resp., E_h) by r_i (resp., r_h). According to Lemma 2, $r_i - r_h = \{p * g + ((j-1)T_i + O_i - O_h) \bmod g\}$, where $g = \gcd(k_hT_h, k_iT_i)$, $p \in Z$. Note that each unique value of $0 \leq (r_i - r_h) < k_hT_h$ may result in a different execution interference of τ_h for the corresponding τ_i's job. However, for $(r_i - r_h) < 0$ or $(r_i - r_h) \geq k_hT_h$, the execution interferences simply repeat the cases for $0 \leq r_i - r_h < k_hT_h$. Therefore, the computation of execution interference between two tasks can be greatly simplified and is outlined in Algorithm 1. The concept of *execution interference* between tasks forms the basis of our proposed approaches to be discussed in the next section.

Algorithm 1 Calculate the execution interference between two tasks

Input: $\tau_i = \{O_i, T_i, D_i, C_i, m_i, k_i\}$, $\tau_h = \{O_h, T_h, D_h, C_h, m_h, k_h\}$, $\Pi_i, \Pi_h, h < i$
Output: $\mathcal{F}_i^h$ //execution interference of τ_h with τ_i
$\mathcal{F}_i^h = 0$;
$g = \gcd(k_iT_i, k_hT_h)$;
for j from 1 to k_i **do**
  if $\pi_{ij} = 1$ **then**
    $x = (O_i + (j-1)T_i - O_h)\ mod\ g$;
    while $x < k_hT_h$ **do**
      F_{ij}^h is calculated according to (4);
      if $\mathcal{F}_i^h < F_{ij}^h$ **then**
        $\mathcal{F}_i^h = F_{ij}^h$;
      end if
      $x = x + g$;
    end while
  end if
end for

4 Our Approaches to the (m,k) Scheduling Problem

In this section, we first present a heuristic technique to improve the (m,k)-patterns obtained by [16]. We then propose a metric that can be used as an objective function in any probabilistic optimization algorithm. Finally, we derive a sufficient condition to predict the schedulability of a task set with given (m,k)-patterns.

4.1 Improving Evenly Distributed (m,k)-Patterns

In Section 2, we know that the algorithm in [16] results in (m,k)-patterns that are not always desirable. We hereby present a heuristic technique to obtain better (m,k)-patterns by judiciously "rotating" the (m,k)-patterns computed by (2). The key idea is to reduce the execution interference between tasks.

As mentioned before, execution interference between tasks can have a significant impact on the schedulability of a task set. It would be very helpful if we know at what instants the maximum execution interference for a given set of (m,k)-patterns may occur. We introduce the concept of *worst-case interference point* to capture this concept.

Definition 6 *A **worst-case interference point** (WCIP) of task τ_i is the beginning instant of a time interval such that the number of mandatory jobs of τ_i is the largest among all time intervals of the same length.*

Based on the above definition, for the (m,k)-patterns defined in (2), time 0 is a *worst-case interference point* since interval $[0, t]$ contains the largest number of mandatory jobs compared with any other interval of the same length. Note that any task, τ_i, has an infinite number of WCIPs for a given (m,k)-pattern and they occur periodically with period $k_i T_i$. If a mandatory job from a lower priority task is released at the same time as one of the WCIPs of some higher priority tasks, the job will apparently suffer the largest execution interference from the higher priority tasks. Intuitively, given a set of (m,k)-patterns, if a WCIP of a lower priority task and those of higher priority tasks concur, it will be more difficult to meet the (m,k) constraints, which is the case for the (m,k)-patterns by [16].

If (m,k)-patterns can be defined such that the WCIPs between tasks are as far apart as possible, the schedulability of the task set would be improved. One way to achieve this is to modify (2) as follows.

$$\pi_{ij} = \begin{cases} 1 & \text{if } j = \lfloor \lceil \frac{((j-1)+s_i)\times m_i}{k_i} \rceil \times \frac{k_i}{m_i} \rfloor + 1 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where $s_i > 0$ and $s_i \in Z$. Note that the new (m,k)-pattern can be viewed as rotating the (m,k)-pattern in (2) right by s_i bits. The new (m,k)-pattern certainly satisfies the (m,k) constraints. Furthermore, we have the following lemma.

Lemma 3 *For τ_i with the (m,k)-pattern defined in (5), the number of mandatory jobs of τ_i is the largest in $[s_i \times T_i, s_i \times T_i + t]$ compared with those within any other interval of the same length t.*

The proof can be readily obtained by applying Lemma 4 in [16] and is thus omitted. According to Lemma 3, by rotating the original (m,k)-pattern defined in (2), we essentially move the first WCIP of task τ_i from 0 to $s_i T_i$. Hence, through careful selection of s_i ($0 \leq s_i < k_i$) values, we can alter the separation between WCIPs of different tasks.

Our problem now becomes determining the value for s_i to separate WCIPs among tasks as far as possible. Since the WCIPs of a task occur periodically, we resort to Lemma 2 in our search for better s_i values. Given task τ_i and the (m,k)-pattern defined in (5), the WCIPs for τ_i is a periodic event with period $k_i T_i$ and initial time $O_i + s_i T_i$. According to Lemma 2, the distance between the closest WCIPS of the two tasks, τ_i and τ_j, is never bigger than $\gcd(k_i T_i, k_j T_j)/2$. Hence, we can select s_i and s_j such that the distance is as close to $\gcd(k_i T_i, k_j T_j)/2$ as possible to reduce the execution interference between the two tasks. For task sets containing three or more tasks, we design a greedy algorithm shown in Algorithm 2.

Algorithm 2 Algorithm for finding rotation values for (m,k)-patterns

```
Input: Task set T = {τ_1, τ_2, ..., τ_n}, where τ_i = {O_i, T_i, D_i, C_i, m_i, k_i}
Output: s_1, ···, s_n //rotation values for each tasks
Ψ = ∅; // Ψ contains the tasks whose s_i values have been determined
while T is not empty do
  τ_i = task in T with the smallest k_i;
  if Ψ ≠ ∅ then
    Ω = Ψ;
    while Ω ≠ ∅ do
      τ_j = task in Ω such that F_i^j is maximum, where F_i^j is defined in Section 3.2;
      g = gcd(k_i × T_i, k_j × T_j);
      if g = 1 then
        remove τ_j from Ω;
      else
        break;
      end if
    end while
    O'_j = O_j + s_j × T_j;
    s_i = l such that 0 ≤ l < k_i and | l × T_i + O_i − O'_j | is nearest to one of (2q+1) × g/2, q ∈ Z;
  else
    s_i = 0;
  end if
  Add τ_i to Ψ;
  Remove τ_i from T;
end while
```

The basic idea of Algorithm 2 is to reduce the worst case response time of mandatory jobs by reducing the execution interference between tasks. Observe that the larger the value k_i is, the more choices task τ_i has for the position of its first WCIP. Hence, among the remaining tasks whose s_i values need to be determined, the algorithm always pick the one having the smallest k_i in its (m,k) constraint. Then, the algorithm selects task τ_j from the tasks whose s_j values have been determined such that the execution interference between τ_i and τ_j is the largest. The s_i value is then set so that the distance between the WCIPs are maximized. Note that in the case when $\gcd(k_i T_i, k_j T_j) = 1$, no matter what the initial positions of WCIPs are, they will eventually meet at some time instant in the future. If this happens, we simply go on to the next task.

Algorithm 2 is quite effective in improving the schedulability of task sets with (m,k) constraints. We will give experimental results later to illustrate this. Furthermore, we have the following theorem.

Theorem 5 *If a task set can be scheduled with the (m,k)-patterns defined by (2), it can always be scheduled with the (m,k)-patterns defined in (5) with s_i determined by Algorithm 2.*

Proof: Consider first the case when the (m,k)-patterns are derived by (2). The first job of each task τ_i is always a mandatory one and has the worst case response time. The work load of the job during $[0, t]$ is as defined in (3), i.e.,

$$W_i(0,t) = \sum_{j \le i} l_{ij} \times C_j, \tag{6}$$

where l_j is the number of mandatory jobs of τ_j ($j \le i$) in $[0, t]$. If τ_i is schedulable, we have

$$W_i(0, t_1) = t_1 \le T_i.$$

Now, let the (m,k)-patterns be rotated by s_i values obtained from Algorithm 2. Similar to the proof for Theorem 3, we only consider the case as shown in Figure 3(b). In Figure 3(b), the work load of τ_i in $[r_{iq}, r_{iq} + t]$ is

$$W_i'(r_{iq}, r_{iq} + t) = \sum_{j \le i} l_j' \times C_j,$$

where l_j' is the number of mandatory jobs of $\tau_j (j \le i)$ in $[r_{iq}, r_{iq} + t]$. Since the (m,k)-patterns are a rotated version of the ones derived from (2), by Lemma 3, we can conclude that $l_j' \le l_j$. Thus $W_i'(r_{iq}, r_{iq} + t) \le W_i(0, t) = t \le T_i$. It follows that τ_i can be scheduled. □

4.2 A Metric for Evaluating (m,k)-patterns in Probabilistic Optimization

Though the algorithm proposed in the previous section is capable of improving the schedulability of task sets employing the (m,k)-patterns derived in [16], there exist cases where no rotating (m,k)-patterns can improve the schedulability. This was demonstrated by the example in Figure 2 in Section 2. In such cases, evenly distributed (m,k)-patterns are not appropriate. We need to find other (m,k)-patterns. Since determining the optimal (m,k)-patterns is NP-hard, a natural contender for solving the problem is a probabilistic optimization approach based on such as genetic algorithms (GA) or simulated annealing (SA), both of which have been shown to be effective in solving many NP-hard problems [8, 17]. GA and SA differ in their mechanism for escaping local minima, but both need an effective objective function to help direct the search process. A major factor to the success of such an approach is the choice of the objective function. We borrow the term *fitness* from GA to refer to the objective function, where a higher fitness value indicates a better solution. In this subsection, we present a fitness function which is quite effective for finding superior (m,k)-patterns.

An ideal fitness function should be able to reflect the fact that using one set of (m,k)-patterns may make the system "easier" to be scheduled than another set. The challenge is how to describe this "easiness". Intuitively, a set of (m,k)-patterns leading to shorter worst case response times for tasks is better. Yet, we have shown in Section 3 that, with arbitrary (m,k)-patterns, finding the worst case response time of a task is NP-hard. As mentioned before, the execution interference suffered by a task directly impacts the schedulability of the task. We hereby propose to use the execution interference between the tasks to formulate the fitness function. Specifically, let the fitness of τ_i be $f(\tau_i)$. Then, we have

$$f(\tau_i) = \frac{T_i}{C_i + \sum_{h=1}^{i-1} \mathcal{F}_i^h}. \tag{7}$$

The denominator in (7) is an estimated worst case work load for τ_i and all the higher priority tasks during any time interval of length T_i. To define the overall fitness value for a task set with some known (m,k)-patterns, we notice that a task set is considered to be unschedulable if any one of its tasks misses the deadline. Hence, the task-set fitness, denoted by $f(\mathcal{T})$, is the minimum among the fitness values of all tasks, i.e.,

$$f(\mathcal{T}) = \min_{1 \le i \le n} f(\tau_i), \tag{8}$$

Given a task set with known (m,k)-patterns, evaluating $f(\tau_i)$ hinges on computing the execution interferences between pairs of tasks, which can be obtained by Algorithm 1.

After the fitness function is obtained, we can apply either a GA or SA approach to search for better (m,k)-patterns. We should point out that the fitness function defined above is only an indicator of the task set schedulability. That is, we cannot guarantee that for any given task sets $\mathcal{T}_1$ and $\mathcal{T}_2$, $\mathcal{T}_2$ must be schedulable if $f(\mathcal{T}_1) < f(\mathcal{T}_2)$ and $\mathcal{T}_1$ is schedulable. However, we have used the fitness function in a GA implementation and the experimental results are extremely encouraging as shown in the next section. In our GA implementation as shown in Figure 5, a gene is defined

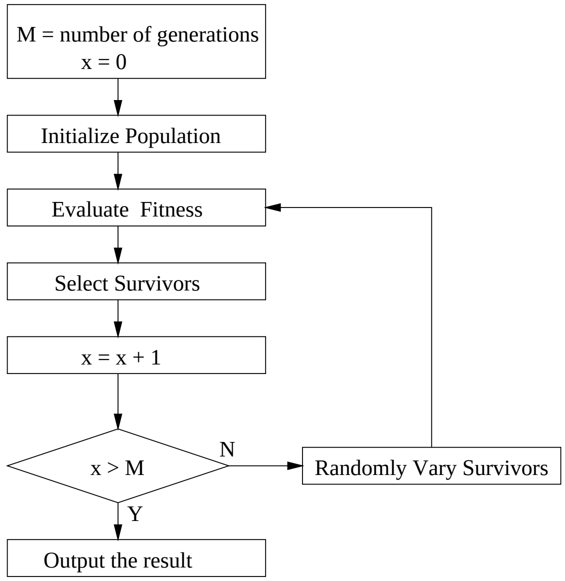


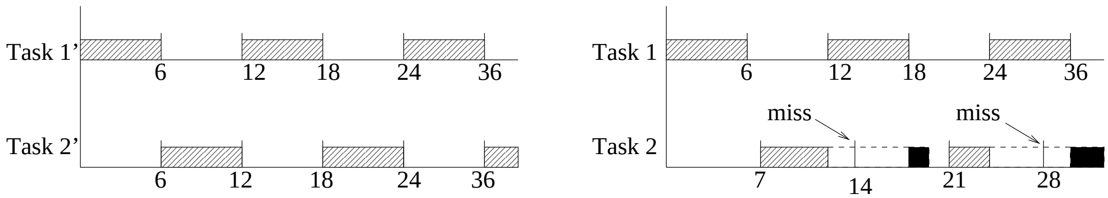
Figure 5: Procedure of genetic algorithm for finding the (m,k)-patterns

as a tuple of a task and its (m,k)-pattern, i.e., (τ_i, Π_i). The reproduction strategy is quite straight forward. The mutation operation selects a gene and changes one bit in the (m,k)-pattern from 1 to 0 and another bit from 0 to 1. The crossover operation selects a cross point for two individuals and swaps their contents to construct two new individuals. More detailed information on GA can be found in [8]. While the effectiveness of our approach is demonstrated in the experiments, how to construct a better fitness function remains an open problem.

4.3 Determining Schedulability for Given (m,k)-patterns

We have proposed two methods to find better (m,k)-patterns for the (m,k) scheduling problem. Yet, we still face the challenge of determining if a task set is schedulable for some given (m,k)-patterns. Answering this question becomes critical when the first job of every task no longer has the worst case response time. In section 3, we have proved that this is an NP-hard problem. Note that a task τ_i with certain k-patterns can be viewed as m_i periodic tasks with period k_iT_i, deadline D_i, and initial offsets $(a_i - 1)T_i + O_i$ (a_i is the index of the mandatory job in τ_i's (m,k)-pattern). One way to deal with this problem is to apply exact analysis [1, 18] for all the possible jobs where the worst case response time may happen which is quite prohibitive for large task sets. In the following, we construct a sufficient condition to test if a task set with known (m,k)-patterns is schedulable. Our goal is to be able to efficiently evaluate such a condition, and hence quickly decide if a set of (m,k)-pattern, derived by an approach such as those described above, can meet the (m,k) constraints. To simplify the problem, we assume that the deadline of a task equals its period.

Our sufficient condition is an extension to the work by Han and Tyan [10]. In [10], a polynomial-time algorithm is proposed to test the schedulability of a hard real-time system scheduled with RMA. The basic idea is to map each task in the task set to a new task such that the new task period is less than or equal to the original period and the computation time remains the same. An additional requirement is that the new task set must be *harmonic*, i.e., any shorter task period must divide any longer task period. It is proved in [10] that if the harmonic task set is schedulable, so is the original task set. However, this is no longer true for a task set with (m,k) constraints. Figure 6 illustrates such an example, where $\mathcal{T} = \{\tau_1, \tau_2\}$, $T_1 = C_1 = 6, T_2 = 7, C_2 = 6$, and $(m_1, k_1) = (m_2, k_2) = (1, 2)$. The corresponding harmonic task set $\mathcal{T}' = \{\tau_1', \tau_2'\}$ with $T_1' = C_1 = 6, T_2' = C_2 = 6$, and $(m_1', k_1') = (m_2', k_2') = (1, 2)$. As shown in Figure 6(a), $\mathcal{T}'$ can be easily scheduled by executing the mandatory jobs alternatively, but $\mathcal{T}$ cannot be scheduled with the same (m,k)-patterns as shown in Figure 6(b).



(a) A harmonic task set is schedulable with 10 as the (m,k)-pattern for Task 1 and 01 for Task 2

(b) A general task set using the same (m,k)-patterns as those in (a) is not schedulable even though the tasks have longer periods

Figure 6: Harmonic task set and its original task set

We derive a sufficient condition that can be applied to tasks with (m,k) constraints. Consider task τ_i in a *harmonic* task set $\mathcal{T}$. Let τ_j has higher priority than τ_j. Then for any mandatory job of τ_i released at t_0, at most $\lceil \frac{T_i}{T_j} \rceil$ mandatory jobs of τ_j occur in $[t_0, t_0 + T_i]$. Since $\mathcal{T}$ is a harmonic task set, so

$$\lceil \frac{T_i}{T_j} \rceil = \begin{cases} \frac{T_i}{T_j} & T_i > T_j \\ 1 & \text{otherwise} \end{cases}$$

Suppose l_{ij} is the maximum number of mandatory jobs from τ_j during any time interval of length T_i. Let

$$W_i = \sum_{j \leq i} (l_{ij} \times C_j) \tag{9}$$

Then, if $\frac{W_i}{T_i} \leq 1$, the total work load by the τ_i's job under consideration and all other higher priority mandatory jobs can be completed in one τ_i's period. Hence, task τ_i is certainly schedulable. For general task sets, we have the following theorem.

Theorem 6 *Given two task sets $\mathcal{T}$ and $\mathcal{T}'$ with $T_i' \leq T_i, C_i' = C_i, m_i' = m_i, k_i' = k_i$, and T_j' divides T_i' if $T_j' \leq T_i'$. With the given (m,k)-patterns, if $\sum_{j \leq i}(l_{ij} \times C_j)/T_i' \leq 1$, where l_{ij} is the* maximum *number of mandatory jobs during any time interval of length T_i', then $\mathcal{T}$ is schedulable.*

Proof: Suppose that the worst case response time of τ_i happens at τ_{iq} as shown in Figure 3(a). Similar to the proof for Theorem 3, we only consider the case as shown in Figure 3(b). In Figure 3(b), we have

$$W_i(r_{iq}, r_{iq} + T_i') = \sum_{j \leq i}(p_{ij} \times C_j),$$

where p_{ij} is the maximum number of mandatory jobs of τ_j during $[r_{iq}, r_{iq} + T_i']$. Since $T_j \geq T_j'$, we have $p_j \leq l_{ij}$, and thus

$$W_i(r_{iq}, r_{iq} + T_i') \leq \sum_{j \leq i}(l_{ij} \times C_j) \leq T_i' \leq T_i.$$

It follows that job τ_{iq} with the worst case response time can be finished before its deadline. Hence, τ_i is schedulable. $\square$

Given a task set with (m,k) constraints, we can apply the algorithm in [10] to find the corresponding harmonic task set, and determine l_{ij} from the given (m,k)-patterns. Then, by Theorem 6, the schedulability of the task set can be tested. A straightforward implementation of our sufficient condition takes $O(n^3klogn)$ time, where $k = \max_i k_i$ and n is the number of tasks. Note that our analysis above is based on the case when $D_i = T_i, i = 1, ..., n$. The result can be extended to the case when $D_i < T_i, i = 1, ..., n$ with the similar approach shown in [10]. How to get tighter sufficient condition without greatly increasing the computational cost is another open problem.

5 Experimental Results

In this section, we present some experimental results to compare the performance of our approaches with that in [16]. For ease of explanation, we use **Alg_Orig** for the algorithm in [16], **Alg_RT** for Algorithm 2 in Section 4.1, and **Alg_GA** for the genetic algorithm approach that employs the fitness function discussed in Section 4.2.

Recall that the goal of our approaches is to select a set of (m,k)-patterns such that they will make the given task set easier to be scheduled. According to Theorem 3, a task set can be scheduled with any set of (m,k)-patterns if it is schedulable with the *deeply-red* (m,k)-patterns. In this case, there would be no benefit to apply the (m,k)-patterns obtained by either [16] or our approaches. Hence, we discard such task sets during our experiments. Moreover, since task sets with low utilization factor values are easier to be scheduled even with (m,k) constraints, a fair comparison needs to study a large spectrum of utilization factor values.

In our experiments, we consider task sets with 5 tasks. The period of each task is randomly selected from a uniform distribution between 10 to 50, and the deadline of each task is assumed to equal its period. The m_i and k_i values are also randomly selected, where k_i is uniformly distributed between 2 to 10, and m_i is uniformly distributed between 1 and k_i. We partition the utilization factor values into intervals of length 0.2. Then, the execution time of each task is randomly selected such that the utilization values of the resulting task sets are uniformly distributed within each interval. To reduce statistical errors, the number of task sets schedulable by at least one of the approaches is no less than 50 within each interval, or at least 5000 different task sets have been generated for the interval. In the genetic algorithm implementation (**Alg_GA**), both the number of individuals and the number of generations are set to 30. To precisely assess the performance of the approaches, we resort to simulation to check the schedulability of a task set for a given set of (m,k)-patterns.

The program is run for 10 times and the average results are collected in Table 1. In our experiments, task sets with utilization values less than 0.8 are all schedulable with their corresponding deeply-red (m,k)-patterns, and none of the task set with utilization greater than 2.0 is schedulable with any of the approaches. Hence, we omit these data from Table 1. In Table 1, columns 2-4 list the average numbers of schedulable task sets by each approach across 10 runs. The columns labeled "Improvement" represent the relative improvements of our two approaches over the approach in [16].

	No. of Schedulable Task Sets			**Improvement(%)**	
Utilization	**Alg_Orig**	**Alg_RT**	**Alg_GA**	**Alg_RT**	**Alg_GA**
0.8 - 1.0	28.3	31.1	31.2	9.89	12.24
1.0 - 1.2	133.7	153.7	151.1	15.96	13.01
1.2 - 1.4	105.6	123.9	127.8	17.32	21.02
1.4 - 1.6	20.1	26.6	36.3	32.34	80.60
1.6 - 1.8	1.6	3.0	6.1	87.50	281.25
1.8 - 2.0	0.0	0.1	0.6	NaN	NaN

Table 1: Experimental results comparing the three approaches

From Table 1, one can conclude that both **Alg_RT** and **Alg_GA** improve the performance of **Alg_Orig**, and the improvements become more significant as the task-set utilization factor values increase. In the experiments, as we expect, a task set is schedulable with **Alg_RT** as long as it is schedulable with **Alg_Orig**. We want to point out that there exist few cases when a task set is schedulable with **Alg_Orig** but cannot be scheduled with **Alg_GA**. However, as shown in Table 1, for a large number of task sets, much more task sets can be scheduled with **Alg_GA**, and in most cases, **Alg_GA** has the best performance among the three approaches in terms of the number of task sets satisfying the (m,k) constraints.

6 Conclusions

In this paper, we address the problem of scheduling task sets with (m,k) constraints using the fixed-priority scheme. Similar to [16], the scheduling approach is to partition the jobs of each task into mandatory or optional jobs. All the jobs are scheduled according to their fixed priorities with the optional jobs assigned the lowest priority. We prove that finding the optimal partition as well as determining the schedulability of the resultant task set are both NP-hard problems. Since traditional hard real-time analysis techniques cannot be readily employed to analyze the behavior of a task sets with (m,k) constraints, we propose a new technique, based on the General Chinese Remainder Theorem, to quantify the interference between tasks. We then propose two approaches to improve the partitions proposed in [16]. Compared with the approach in [16], our approaches produce better partitions for reducing the interference among mandatory jobs and thus better explore the (m,k) constraints in overloaded systems. We prove that our solution space is a super set of that in [16]. Furthermore, for a task set with arbitrary (m,k)-patterns, whose worst case response time cannot be easily identified, we propose a sufficient condition which takes only polynomial time to predict its schedulability. Experimental results show that the improvements achieved by our approaches are quite significant when the utilization factors of task sets are relatively large.

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