Stochastic Based Extended Krylov Subspace Method for Power/Ground Network Analysis

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Abstract

In this paper, we present a novel stochastic simulation approach based on extended Krylov subspace method for on-chip power grid analysis. The new method performs the analysis by using random walk in a stochastic manner. But different from the existing random walk method, the moments of the circuits are computed and extended Krylov Subspace (EKS) method is used to calculate the responses in frequency domain. The new method can compute the transient responses in a local manner, which is in contrast to the existing random walk method [12], thus improves the existing frequency-domain random walk method [4] by using extended Krylov subspace method. The resulting method is more numerically stable and faster than existing random walk methods. Experimental results demonstrate the advantages of the proposed method, called rwEKS, over EKS method for localized power grid analysis.

1 Introduction

As technology continues to scale, VLSI design has suffered severe Power/Ground (P/G) supply voltage degradation. This is because: (1) technology scaling results in decreased interconnect width and increased interconnect resistance in a P/G supply network, (2) increased device density leads to increased supply current density on a chip, and (3) a higher clock frequency leads to more significant inductance effect. At the same time, supply voltage continues to decrease in modern technologies, which results in a decreased noise margin for signal transition, and makes transistor more vulnerable to supply voltage degradation.

Many research works have been done on efficient simulation of Power/Ground networks to capture electrical behavior characteristics of a P/G supply network. Multigrid-like [5, 8], hierarchical [14] [6] or partition-based [7] approaches help to improve scalability of P/G network analysis by providing a global abstraction of a P/G network.

Krylov subspace based methods have been dominant in frequency domain analysis for signal propagation interconnect analysis [9]. Extended Krylov Subspace (EKS) [13, 6] is an efficient simulation technique and is widely used for P/G network analysis. It directly computes the orthogonalized moments of the response in a gradually increasing way when many active sources are functioning simultaneously. Thus, unlike PRIMA [9], its runtime is independent of the number of ports (active sources) of the network.

Furthermore, because the supply voltage degradation of a P/G node is largely determined in its neighborhood region, i.e. from the node to a nearby P/G supply pad [12]. Random walk techniques that exploit locality of supply voltage degradation

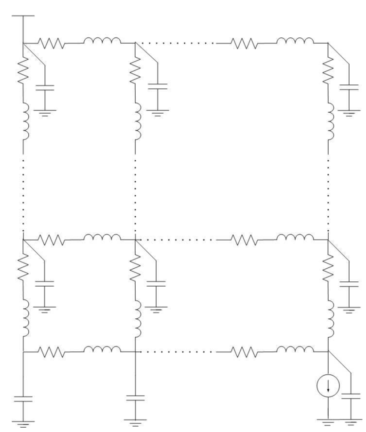

Figure 1: Power ground supply network

was used to analyze practical size P/G supply networks with a large number of P/G supply pads [3]. Its runtime is more dependent on its path length, rather than the P/G network size. However, conventional random walk techniques are only applied to DC analysis in pure resistive networks, as well as to time-domain transient analysis in RC networks [12]. In this paper, we present a new random walk based moment computation technique for stochastic power grid network analysis (rwEKS). The new method allow localized computation of transient responses in a numerical efficient way. In contrast, existing work like [12] can't compute transient responses in a localized way as every node voltage response has to be computed. The new method also compare favorably wit existing extended Krylov subspace as rwEKS allow a few nodes to be computed instead of the whole circuit. Experimental results confirm the effectiveness of the proposed method.

2 Background

2.1 Basic Concepts and Notations

A P/G supply network is modeled as a distributed RLC(K) network which includes interconnect resistors and ground capacitors. Power/Ground pads are modeled as DC voltage sources, while active devices which draw supply currents are modeled as time-varying current sources. Each active device injects a supply current, of maximum envelope current waveform [1], during signal transition in a "timing window", i.e. the time frame bounded by the earliest and the latest signal arrival times in timing analysis.

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The P/G supply network problem is formulated as follows: Given an RLC(K) netlist of a P/G supply network with a subset of nodes of DC supply voltage sources, and a subset of nodes with supply current sources, the problem is to find the voltages of a subset of the desired observing nodes.

The following notations are used in this paper.

- (p,q) = edge between nodes p and q,
- E = set of edges of P/G netlist,
- C_q = ground capacitance at node q,
- G_{pq} = conductance between nodes p and q,
- L_{pq} = inductance between nodes p and q,
- V_q = voltage of node q,
- I_q = current of the current source at node q,
- $\overline{I_q} = sC_qV_q + I_q$ = total ground current of node q,
- $m_i(q)$ = the i-th order moment of the voltage of node q,
- $m_i(I_q)$ = the i-th order moment of the source current I_q.

2.2 Extended Krylov Subspace Based Analysis

Krylov subspace based reduction methods have been well-accepted and in interconnect model [9]. But the methods are inconvenient for supply network analysis, due to the presence of a large number of inputs (supply current excitations) and outputs (potential supply voltage degradation nodes). To mitigate this problem, extended Krylov subspace method was proposed [13, 6]. The idea is to perform the reduction on both model and input modeled as piecewise linear signals. The computation is intended to be independent of the number of ports (active sources) in the network.

Instead of computing the vectors of the n-th order moments for explicit moment matching, extended Krylov subspace method constructs a modified Krylov subspace by orthonormalizing the moment vectors with current sources. For a RLC network,

$$G + sC = Bu \tag{1}$$

where G is conductance matrix, C is storage element matrix. B is position matrix. The response moments can be represented as

$$(G+sC)(m_0+m_1s+m_2s^2+... = B(u_0+u_1s+u_2s^2+...) \tag{2}$$

The EKS Algorithm 1 shown below essentially perform the orthonormalization on the responses moments m_i.

Algorithm 1: Extended Krylov Subspace Method

Input: Circuit of G, C, B, u

Output: projection matrix $V = \bar{r}_0, \bar{r}_1, ...\bar{r}_n$, where span $(V) = span(m_0, m_1, ...m_n0)$, m_i is the moment of the circuit with current source.

1. $b_0 = B\bar{u}_0$, $m_0 = G^{-1}b_0$, $\hat{r}_0 = \alpha_0 m_0$
2. for $i = 1 : d$
3. $\quad r_i = G^{-1}(\Pi_{j=0}^{i-1}\alpha_j B\bar{u}_i - C(\hat{r}_1 + \alpha_{i-1}\Sigma_{j=0}^{i-1} h_{i-1,j}\hat{r}_j))$
4. $\quad$ for $k = 1 : i-1$
5. $\quad\quad h_{i,k} = \hat{r}_k^T r_i$
5. $\quad \bar{r}_i = r_i - \Sigma_{j=0}^{i} h_{i,j}\hat{r}_j$
6. $\quad$ if $norm(\bar{r}_i) \leq \varepsilon$ break;
7. $\quad$ else $\hat{r}_i = \frac{\bar{r}}{norm(\bar{r})}$, $\alpha_i = \frac{1}{norm(\bar{r})}$

After that we can perform the congruence transformation by $\bar{G} = V^T GV$, $\bar{C} = V^T CV$, $\bar{B} = V^T B$, we end up with reduced system.

$$\bar{G} + s\bar{C} = \bar{B}u \tag{3}$$

Transient simulation can be carried out on (3), which be very efficient due to reduced circuit matrices.

3 Krylov Subspace Based Stochastic Analysis Method

3.1 Review of Random Walk in a Resistive Network

P/G network analysis can be performed by traditional direction solution using LU decomposition [11]. However, direct solvers suffer computational efficiency and scalability problems for very large problem sizes. A more efficient approach is to apply Monte Carlo methods to solve partial differential equations (PDEs), e.g., random walk [2].

Random walk has been applied to DC analysis in purely resistive networks and time domain transient analysis in RC networks [12]. For example, in a purely resistive P/G network (zero inductances and capacitances), for a node q, with conductance G_{pq} between q and each of its neighboring node p, and a current source I_q between q and the ground, Kirchoff's current law states that the ground current I_q and voltage V_q of node q are as follows:

$$V_q = \frac{\Sigma_{(p,q)\in E} G_{pq}V_p - I_q}{\Sigma_{(p,q)\in E} G_{pq}} \tag{4}$$

This is the finite-difference form of a boundary value problem of partial differential equations. Each node voltage is associated with the nearby node voltages, as well as the known node voltages satisfying the boundary conditions [5].

One of the Monte Carlo methods of solving PDEs, i.e., random walk, is described as follows [12]. A traveler pays an amount $A(q) = \frac{I_q}{\Sigma_{(p,q)\in E} G_{pq}}$ (e.g., for lodging) at a node q, with a probability $Prob(p,q) = \frac{G_{pq}}{\Sigma_{(p,q)\in E} G_{pq}}$ of going to an adjacent node p, until he reaches a node of V_{dd} voltage source (home), where he stays and receives a reward (which equals the V_{dd} voltage). The average net gain (the reward minus the costs) of the trip approaches the voltage at node q (Algorithm 2). Such a random walk game avoids prohibitive full-scale analysis and exploits locality of the problem (since most of the traversals are at neighboring nodes). However, accuracy improves with increased number of random walk process [12].

The problem for the random walk method [12] is that for solving for transient response, the voltage responses of every node need to be computed. So the method loses its advantage. This problem is partially mitigated by recent stochastic moment marching method (SMM) [4], where moments are computed in local way via random walk method. But explicit moment matching was used to compute the responses, which suffer numerical problem. In this way, we improve SMM method by applying more stable Krylov subspace method numerically.

Algorithm 2: Random Walk in a Resistive Network

Input: Resistive netlist, boundary nodes with known voltages

Output: Voltage of the observation node

1. Starting from the observation node
2. While(not reaching a boundary node)
3. $\quad$ Pay $A(q)$ at node q
4. $\quad\quad$ Walk to an adjacent node p with transition probability $Prob(p,q)$
5. Gain V_b of the voltage at boundary node b
6. Return the net gain

3.2 New Krylov Subspace Random Walk Based Analysis Algorithm

Random walk based Stochastic Moment Computation (SMC) problem is discussed in [4]. For a node q in an RLC network, with conductance $G_{pq} = (R_{pq} + sL_{pq})^{-1}$ between node q and a neighboring node p, and a total ground current $\overline{I_q} = sC_q V_q + I_q$ through a ground capacitor C_q and a current source I_q at node q, Kirchoff's current law states:

$$\sum_{(p,q)\in E} \frac{V_q}{R_{pq} + sL_{pq}} = \sum_{(p,q)\in E} \frac{V_p}{R_{pq} + sL_{pq}} - sC_q V_q - I_q \quad (5)$$

Expanding V_p, V_q, and I_q into moments $m_i(p)$, $m_i(q)$, and $m_i(I_q)$ respectively, we have

$$\frac{1}{R_{pq} + SL_{pq}} = \sum_i (-1)^i \frac{i! L_{pq}^i}{R_{pq}^{i+1}} s^i$$

$$\sum_{(p,q)\in E} \frac{1}{R_{pq}} m_j(q) = \sum_{(p,q)\in E} \Big(\frac{1}{R_{pq}} m_j(p) + \sum_{i=1}^{j} (-1)^i \Big(\frac{i! L_{pq}^i}{R_{pq}^{i+1}}\Big) (m_{j-i}(p) - m_{j-i}(q))\Big) - C_q m_{j-1}(q) - m_j(I_q) \quad (6)$$

Rewriting (6) as follows to enable a random walk game.

$$m_j(q) = Prob(p,q) m_j(p) + A(q) \quad (7)$$

where

$$Prob(p,q) = \frac{R_{pq}^{-1}}{\sum_{(p,q)\in E} R_{pq}^{-1}}$$

$$A(q) = \sum_{(p,q)\in E} \frac{1}{R_{pq}^{-1}} \sum_{i=1}^{j} (-1)^i \Big(\frac{i! L_{pq}^i}{R_{pq}^{i+1}}\Big)(m_{j-i}(p) - m_{j-i}(q)) - \frac{C_q m_{j-1}(q) + m_j(I_q)}{\sum_{(p,q)\in E} R_{pq}^{-1}}$$

In such a random walk game, a traveler pays an amount of $A(q)$ for lodging at node q, with a probability $Prob(p,q)$ of going to an adjacent node p, until he reaches home or a V_{dd} node. At each step, we have $m_j(q) = m_j(p) + A(q)$, where $A(q)$ includes low order moments and is known if we compute moments in an increasing order. Averaging over a number of random walks with $Prob(p,q)$ gives the same moment $m_j(q)$ as in (6).

Algorithm 3: rwEKS algorithm description

Input: RLC P/G network, V_{dd} nodes V, current sources S
Output: Orthogonalized moments of nodes R

1. For $iteri = 1 : iternum$
2. For each current source node $s \in S$
3. Random walk from s to node $v \in V$ following $Prob(p,q)$
4. For each node q in the path
5. For each moment order j
6. Compute $m_j(q)$ according to (6)
7. Compute the average moments of each node
8. For $iteri = 1 : iternum$
9. For each unvisited node $s \in R$ whose moments are unknown
10. Random walk from s to node v whose moments are known
11. For each node q in the path
12. For each moment order j
13. Compute $m_j(q)$ according to (6)
14. Compute the average moments of each node;
15. Using EKS method to compute the response on the selected nodes.

The new algorithm is summarized in **Algorithm 3**. Basically, we compute the response moments using random walk method based on (7). After this, EKS is used to compute the project matrix by orthonormalizing the response moments and compute the responses for the nodes we are interested in. One observation is that taking average moments over different random walk processes does not improve solution accuracy. The parameter $iternum$ of algorithm 3 is not the larger the best. It shows that the value between 5 and 10 is required to achieve the best results. This feature also prevents the accumulation of numerical instability during moment computations.

Table 1: CPU time comparison (in seconds) of EKS and rwEKS.

	# of nodes in P/G circuits	EKS (s)	rwEKS (s)	Speedup
1	1720	1	2	0.5
2	4900	998	53	18.8
3	6400	2255	72	31.3
4	8100	5502	121	45.5
5	10201	11783	334	35.3
6	14400	20685	483	42.8

4 Experimental Results

We implemented the proposed rwEKS algorithm and EKS algorithm with C++ language in a Linux system with dual Intel Xeon CPUs with 3.06Ghz and 1GB memory. Five P/G circuits are used for testing purpose. The largest circuit has 14400 nodes while the smallest one has 1720 nodes. Each circuit is a mesh grid, where each grid segment consists of a number of resistors and capacitors. Each grid node is attached with possible voltage and current sources. The initial power/ground network is stabilized at the supply voltage before current sources are activated. Upon activation, a current source injects a triangle waveform to model the toggling of a transistor [10]. Note we can apply any piecewise linear waveform. But to simplify the experiments, we only use triangle waveforms. The time domain and s domain functions for a triangle waveform current are as follows:

$$I(t) = \begin{cases} \frac{t}{T_r} I_0 & 0 \le t \le T_r \\ (2 - \frac{t}{T_r}) T_0 & T_r \le t \le 2T_r \end{cases}$$

$$I(s) = I_0 \sum_{i=2}^{n} (-1)^i \frac{2^i - 2}{i!} T_r^{i-1} s^{i-2} \quad (8)$$

Note that the current source has a transition time $T_r = 1ns$ and a peak current of $1mA$.

We first compare the performance of proposed algorithm with EKS algorithm in each circuit. We randomly select 1000 nodes and compute their moment with orthogonalization results. The CPU time to the five circuits are listed in Table 1. From Table 1, we can see that for circuit 1 with 1720 nodes, EKS requires about 1 second to complete, while rwEKS takes about 2 second to complete. However, for the number circuit 6 with 14400 nodes, EKS requires 20685 seconds to finish in comparison to rwEKS that requires only 483 seconds to finish. The speedup of the proposed algorithm is about two order of magnitudes over EKS. Experimental results clearly show that the proposed rwEKS have clear advantage over EKS method due to its locality.

Furthermore, the order reduced circuits are projected back to the original circuits for accuracy comparison. We arbitrarily select circuit nodes and compare the simulation results between the original circuit and the order-reduced circuits obtained by

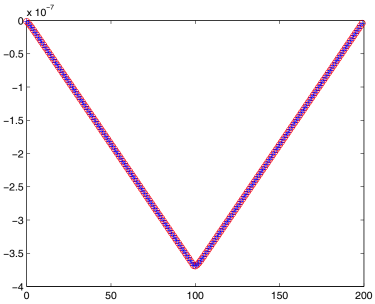

Figure 2: Waveform of a node produced by EKS algorithm.

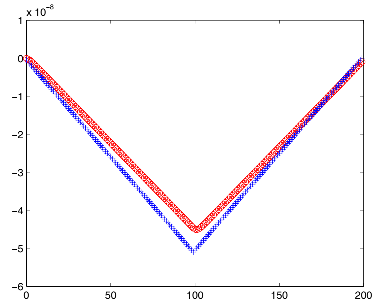

Figure 3: Example 1 - waveform of a node produced by rwEKS

EKS and rwEKS respectively. We observe that the waveforms of all selected nodes from EKS match very well with the ones from the original circuits. A typical waveform of a randomly selected node from EKS is shown in Fig. 2.

The waveforms of all selected nodes from rwEKS do not match those of the original circuits exactly as EKS does. Three typical waveforms for comparison are shown in Fig. 3, Fig. 4 and Fig. 5, respectively. As described in the previous section, the number of times visiting a given node may not necessarily improve the accuracy and stability in computation of moments at that node. Thus, the total runtime of our random walk based rwEKS algorithm is mostly dominant by the orthogonalization procedure.

5 Conclusion

In this paper, we have proposed a new stochastic simulation approach based on extended Krylov subspace method for on-chip power grid network analysis. The proposed method, called rwEKS, targets at simulating a small portions of power grid networks in a sampling based way. It applies improved random walk method to compute the circuit moments in a localized way and extended Krylov subspace to improve the numerical efficiency. Experimental results show that the rwEKS has clear an advantage over the existing EKS method for solving small portion of large power grid networks due to its locality property.

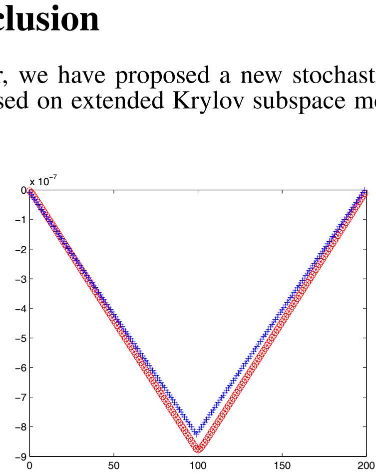

Figure 4: Example 2 - waveform of a node produced by rwEKS

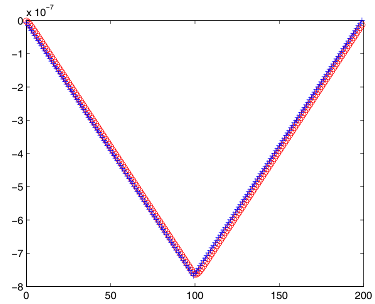

Figure 5: Example 3 - waveform of a node produced by rwEKS

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