Volterra Series Based Signal Processing on Integrated Communication Systems

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Abstract— **In this paper, we present a novel behavior frequency domain modeling in application to modern communication systems to describe the third order inter-modulation distortions and harmonic distortions in high frequency signal processing environments based on Volterra series. By using model order reduction techniques, we are able to speed up the simulation time. Two basic model structures including a frequency shaping block and a resistive nonlinear gain block are analyzed. A compositional model based on the proposed two basic models is presented. Different cascade strategy is also discussed. The proposed compositional model achieves simulation results within 6% error on average in comparison with the results from Spectre simulator.**

Keywords - Volterra series; Inter-modulation distortion; Harmonic distortion ; Model order reduction

I. INTRODUCTION

In the signal processing of communication systems, efficient behavioral models are required to evaluate the performances and reliability of the systems. When considering Radio Frequency (RF) circuits, non-linear effects become more involved, particularly by using the Volterra theory. From a practical point of view, direct implementation of Volterra series to behavioral simulation starting from raw simulation data is a time-consuming task. A more structural method to generate a behavioral model that exhibits a consistent high frequency behavior may be based on simple models whose structure accounts for RF non-linear effects. A straightforward example may be represented by a non-linear characteristics followed by a frequency shaping linear filter. Such a model in communication system might actually give accurate results for both inter-modulation and harmonic distortions at high frequency. However, as the model complexity increases, the parameter extraction based on simulation could be extremely difficult and time-consuming. Therefore, model order reduction techniques are employed to speed up the simulation process. Model order reduction techniques have been widely applied in the fast simulation of large linear and nonlinear systems, such as VLSI interconnect circuits [1][2][3], high speed clock network [4], nonlinear analog RF circuits and MEMS systems [1]-[4] etc.

The goals of this paper are to present proper structures to simulate non-linearity of the signal processing in communication systems at the behavioral level. For example, some mixers can be analyzed and modeled by using Verilog-AMS or Simulink [5][6]. However, the accuracy, speed, and limits of the models have to be analyzed. An important aspect to consider in signal processing is the efficiency of extraction of these models from simulation data. The second goal of this paper is to set up a flexible and efficient simulation strategy to extract behavioral model parameters directly from the circuit netlists.

II. DESCRIPTION OF NONLINEARITY

A. *Power Series*

Power series, also referred as Taylor series, is the most widely used model to describe the nonlinear transfer function of an analog circuit. It represents nonlinearity in a polynomial form [5], such as

$$S_\circ = a_1 S_i + a_2 S_i^{\,2} + a_3 S_i^{\,3} \dots\dots a_n S_i^{\,n} \tag{1}$$

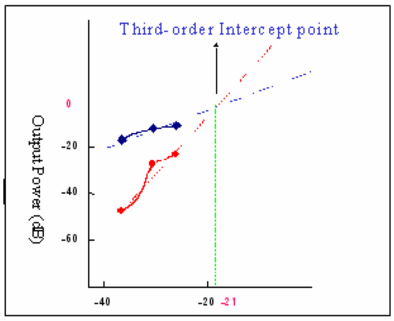

Figure-1: The 3rd order intercept point

At low frequency, the parasitic capacitances and inductances are negligible. Hence, the circuit contains only resistive nonlinearity and is a memory-less system. It means that the instant response of the circuit does not depend upon its previous history. In power series, all the coefficients $a_1, a_2, \dots a_n$ are frequency independent, which lead to some simple but useful mathematical relations between distortions magnitude and input signal level [5][6]. Because all the nonlinear coefficients in a power series are frequency independent, the model can not describe the frequency dependency of the distortions. An example is shown in Figure 1 to illustrate that distortions indeed varies as frequency varies. In order to model high frequency nonlinearity, one has to use Volterra series based models [7].

B. *Volterra Series*

Volterra series [7] is a generalization of power series. It does not assume that the coefficients are frequency independent. As the amplitude of the input increases, the magnitude of the higher order components will increase more quickly than the lower order components. For weakly nonlinear circuits in signal processing that is excited by small signals, usually only the first three terms are retained. The nonlinearity approximated by the linear behavior induces harmonic and intermodulation components. As we know that the Volterra series for a circuit is generally represented as a summation of nth order operators.

$$S_o = H_1(j\omega)\circ S_i + H_2(j\omega_a, j\omega_b)\circ S_i^2 + H_3(j\omega_a, j\omega_b, j\omega_c)\circ S_i^3 + \dots \quad (2)$$

In the Volterra series, every coefficient is a function of frequency. The Volterra operator " ∘ " indicates the times of the magnitude of the coefficient (a complex number) and the shift of the phase by an angle of the complex number. From a mathematical point of view, the quantities in Eq (2) $H_n(j\omega_a, j\omega_b, j\omega_c \cdots\cdots j\omega_n)$ are n-dimensional Fourier transforms of the Volterra kernels [7].

C. *Measuring Voltage series Coeffcients (kernels)*

Direct measuring of Volterra coefficients is inherently difficult because the high order terms are so small that numerical noise often destroys the approximated results. Another factor that affects the accuracy of measurement is that the contribution of higher order terms to lower order distortions. Therefore, in order to make a good estimation of the distortion, we need to make the desired coefficients measureable in the dominant terms at that frequency and exclude the contributions of other coefficients as much as possible.

For example, if we want to measure H (-j ω_1, j ω_2, j ω_3), then we need to measure the 3rd order inter-modulation at that frequency, any contribution from other coefficients should be avoided. It means we need to carefully choose frequencies of input signal pair, so that the distortion terms are at distinctive frequencies [7]. For instance, if we choose a pair of input signals at 6MHz (f1) and 8MHz (f2), then one of the third order inter-modulation components is at frequency 10MHz (2f2-f1).

However, such measurement includes the contributions from higher order terms at frequency (3f1–f2), which introduce errors into our measurement. Usually, we choose two prime integer frequencies, for example 5MHz and 7MHz to avoid this kind of problem. Though exhaustive simulations over a large number pairs of inputs in the desired frequency range are theoretically possible, and simple interpolations methods may also work in the nearby frequencies, it is still desirable to structurally explore the Volterra-series-based models. Because of the structure simplicity, we may be able to predict and estimate the higher order distortions or coefficients based on relatively fast simulations.

III. BASIC BEHAVIORAL MODELS

In this section, we propose two basic behavioral models for weakly nonlinear circuits. The nonlinearity can be well-described by the first three terms in the Volterra series. It implies that all higher order terms contributed to distortions are negligible. The basic models are structured to consist of a resistive nonlinear gain block and a frequency shaping block (*e.g.* linear passive filter).

The idea is that we orthogonalize the frequency dependent nonlinearity into two independent behavior blocks. One is the frequency independent nonlinear gain block, which models the nonlinearity dependency on signal levels (amplitudes). The other is the frequency shaping block, which only captures the behavior of frequency dependency of the nonlinearity of the signal processing.

A. *The Proposed Model-A*

In the proposed model, a resistive nonlinear gain block is cascaded before a frequency shaping block with transfer function x(j ω). Its block diagram is shown in Figure 2. The nonlinear gain block includes the transfer function.

$$S_\circ = a_1 S_i + a_2 S_i^{\,2} + a_3 S_i^{\,3} \quad (3)$$

$$S_{\circ x} = x(j\omega)\circ S_\circ$$

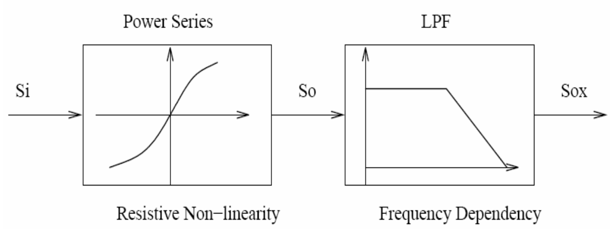

Figure-2. Block diagram of proposed model-A

Thus, the amplitudes of the harmonic terms are:

$$fundamental = (a_1 S_1 - \frac{3}{4} a_2 S_1^{\,3})\,|x(j\omega)| \quad (4)$$

$$3rdOrderHD \quad = \frac{1}{4} a_3 S_1^{\,3} |x(j3\omega)|$$

Assuming $x(j\omega)$ is normalized to its maximum value, the model says that the 3rd order harmonics distortions (HD) component due to an input signal at frequency *f* should have the same frequency response as the fundamental frequency response at 3*f*. The amplitude of the 3rd order inter-modulation distortion (IMD) at frequency $(2f_2 - f_1)$ is:

$$3rdOrderIMD \quad = \frac{3}{4} a_3 S_1 S_2^{\,2} |x(j2\omega_2 - j\omega_1)| \quad (5)$$

If $(\omega_2 \approx \omega_1)$ holds, which is usually true, because the interference from adjacent channels is practically a major concern, then we may conclude

$$3rdOrderIMD \quad = \frac{3}{4} a_3 S_1 S_2^{\,2} |x(j\omega_1)| \quad (6)$$

Thus, we have predicted that the 3rd order inter-modulation has a similar frequency response as the fundamental frequency response.

B. The Proposed Model-B

In the proposed model B, a resistive nonlinear gain block is cascaded after the frequency shaping block with transfer function $y(j\omega)$. Its block diagram is shown in Figure 3.

$$S_{\circ} = a_1 S_i + a_2 S_i^{\,2} + a_3 S_i^{\,3} \quad (7)$$
$$S_{\circ y} = y(j\omega) \circ S_{\circ}$$

We can calculate the amplitudes of the harmonics terms as follows:

$$fundamental = a_1 S_1 \left|y(j\omega)\right| + \frac{3}{4} a_3 S_1^{\,3}) \left|y(j\omega)\right|^3 \quad (8)$$
$$\approx a_1 S_1 \left|y(j\omega)\right|$$
$$3rdOrderHD \quad = \frac{1}{4} a_3 S_1^{\,3} \left|y(j\omega)\right|^3$$

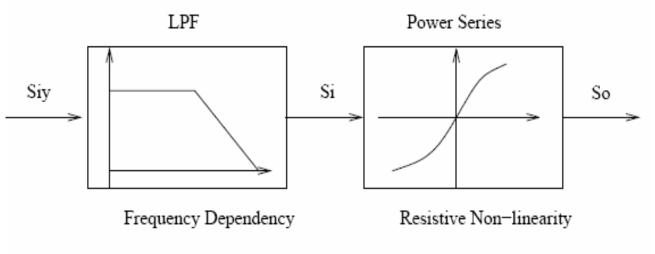

Figure-3 Block diagram of the proposed model – B

The approximation in the fundamental response is based on the assumption that the weak nonlinearity in the signal processing and distortion terms are determined by lower order coefficients. The result shows that the fundamental frequency response is linearly proportional to $y(j\omega)$. Also, the 3rd order inter-modulation distortion at frequency ($2f_2$-f_1) is denoted as:

$$3rdOrderIMD \quad = \frac{3}{2} a_3 S_1 S_2^{\,2} \left|y(j\omega)\right|^3 \quad (9)$$

It means that the frequency response of the 3rd order inter-modulation component is proportional to $y(j\omega)^3$. As compared to the fundamental frequency response, we noticed that there is a correlation between the higher order distortion terms and fundamental frequency responses. This relation suggests that we could use the fundamental frequency response to estimate the frequency response of higher order distortions, *i.e.* the proposed 3rd order inter-modulation components in this paper.

IV. COMPOSITIONAL BEHAVIORAL MODEL

The proposed model – C is a compositional model based on the first two basic models, A and B. It consists of two resistive nonlinear gain block cascaded before and after a frequency shaping block with transfer function. $X(j\omega)$

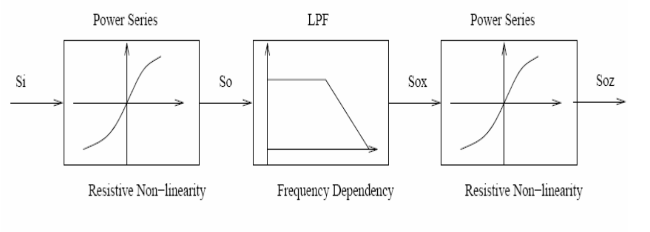

Figure-4. Block diagram of proposed model – C

After doing some calculation, we conclude:

$$S_{\circ} = a_1 S_i + a_2 S_i^{\,2} + a_3 S_i^{\,3}$$
$$S_{\circ x} = x(j\omega) \circ S_{\circ} \quad (10)$$
$$S_{\circ z} = c_1 S_{\circ} + a_2 S_{\circ}^{\,2} + a_3 S_{\circ}^{\,3}$$

Its 3rd order term is:

$$c_1 a_3 x(j\omega_a, j\omega_b, j\omega_c) + 2c_2 a_1 a_2 x(j\omega_a) x(j\omega_b, j\omega_c)$$
$$+ c_3 a_1 x(j\omega_a) x(j\omega_b) x(j\omega_c)] \circ S_i^3 \quad (11)$$

The 3rd order inter-modulation distortion at frequency $(2\omega_1 - \omega_2)$ is:

$$c_1 a_3 x(j2\omega_1 - j\omega_2) + c_3 a_1 x(j\omega_1) x(j\omega_1) x(j\omega_2) +$$
$$2c_2 a_1 a_2 \frac{1}{3}[x(j\omega_1 - j\omega_2) + x(-j\omega_2) x(j\omega_1 + j\omega_1) \quad (12)$$
$$+ x(j\omega_1) x(-j\omega_2 + j\omega_1)]]$$

Assume $\omega_2 \approx \omega_1$, then terms with all the frequencies at $(\omega_1 - \omega_2), (\omega_1 + \omega_1), (-\omega_2 + \omega_1)$ have fallen out of band. 3rd order IMD term is then simplified to:

$$c_1 a_3 x(j\omega_1) + c_3 a_1 x(j\omega_1) x(j\omega_1) x(j\omega_1) \quad (13)$$

Hence, the amplitude of 3rd order IMD approximately equals to

$$c_1 a_3 \left|x(j\omega_1)\right| + c_3 a_1 \left|x(j\omega)\right|^3 \quad (14)$$

It means that the frequency response of 3rd order IMD is a weighted sum of the fundamental frequency response and a power of three to the fundamental frequency response.

Furthermore, in order to measure the weights (coefficients), we could set up simulations for two pairs of input frequencies as follows:

$$\left|\alpha(j\omega_1)\right| + \beta\left|x(j\omega_1)\right|^3 = IMD\,3(\omega_1)$$
$$\left|\alpha(j\omega_2)\right| + \beta\left|x(j\omega_2)\right|^3 = IMD\,3(\omega_2) \quad (15)$$

One frequency pair could be set near f_0 the center frequency of the band, or the low frequency if it is has a low pass frequency shape. Then $x(j\omega_0)$ approximately equals to 1, because $|x(j\omega)|$ is normalized to its maximum value in our proposed models. After that, we could derive the following expressions for α, β

$$\begin{cases} \alpha = \dfrac{IMD\ 3(\omega_0)|x(j\omega_1| - IMD\ 3(\omega_1)}{|x(j\omega_1|^3 - |x(j\omega_1)|} \\ \beta = \dfrac{IMD\ 3(\omega_0)|x(j\omega_1| - IMD\ 3(\omega_1)}{|x(j\omega_1| - |x(j\omega_1)|^3} \end{cases} \quad (16)$$

Consider the circuit function output from model A and B, we use Moment Matching Model order reduction [8] (Pade approximation) to speed up. The approximation method used here is actually a specific type of a well-known rational function approximation -- Pade approximation. The definition of Pade approximation [8] is as follows. Assume a rational function

$$Rn,m(x) = \frac{Bn(x)}{Am(x)} \quad (17)$$

where Bn(x) and Am(x) are polynomials of order n and m, respectively. *Rn,m(x)* is a Pade approximation of type [n/m] to a function called F(x), if the first (n+m+1) terms of its Taylor expansion equal to the Taylor expansion of F(x). The main idea is that we only keep parts of moments of original function here in order to speed up as shown in Fig 5. This model order reduction technique may apply to the proposed Model A, B, and C.

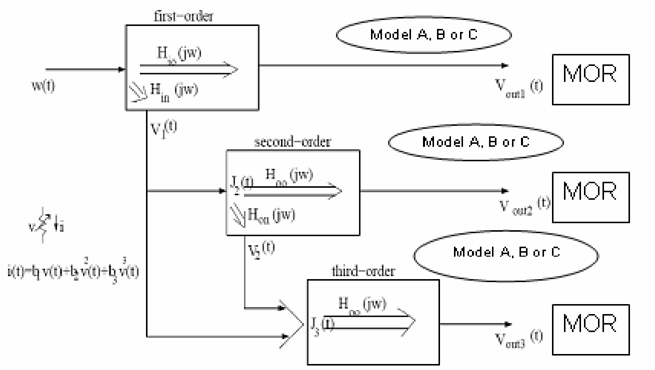

Figure-5. Reduced order modeling system to handle a nonlinear circuit and its first, second and third order Volterra circuits with linear transfer functions.

V. EXPERIMENTAL RESULTS

The simulations are based on wideband IF double conversion CMOS integrated receiver and a CMOS Gilbert double-balanced mixer with center frequency at 2GHz. The comparison results are showed in the plots and tables below. The fundamental frequency response is simulated by performing a PAC analysis on the RF input, with a large signal but no RF signal present. The harmonic distortions are simulated by performing a periodic steady state analysis with fundamental frequency equals the frequency of the RF signal. A small amplitude signal 10mv is applied to the RF input. The inter-modulations of the signal processing are simulated by performing a PDISTO analysis with two equal amplitude signals applied to the RF input. The estimations of the 3rd order IMD are based on the frequency response of the fundamental, and a reference 3rd order IMD at lower frequency. The 3rd order IMD is chosen to be the key performance metrics because other order inter-modulations terms are either too small or out of band. In the mixer, we assume the local oscillator is ideal and does not contribute to the distortions at the output.

Our test results show that Model-A gives relatively accurate 3rd order harmonic distortions in signal processing, compared to the Model-B. But the two curves still track each other, and have the same envelope with similar shape. Also, Model-A provides less accuracy on the 3rd order inter-modulation distortion in comparison to the results from Model-B. The error is about 3~4dBm. The Model-B gives us relatively accurate estimations of the 3rd order inter-modulation component. The error is about 1~2 dBm for low input signal levels and about 3~4dBm for high input signal levels. Since Model-C gives the most accurate results at low input signal levels, we only put the results of Model-C with model order reduction technique here.

Table-1 shows the Monte Carlo simulation based on 100 random inputs. The error is less than 1 dBm. Even at relative high input signal levels, the error is still within 1~2dBm. Obviously, in order to extract and calculate the parameters of Model-C, we need to perform PDISTO analysis twice at two different frequency pairs. It means that its cost is approximately twice as the first two basic models. Harmonic distortions are compared with Wiener module, and Model-C with model order reduction gives more accurate and less time consuming results.

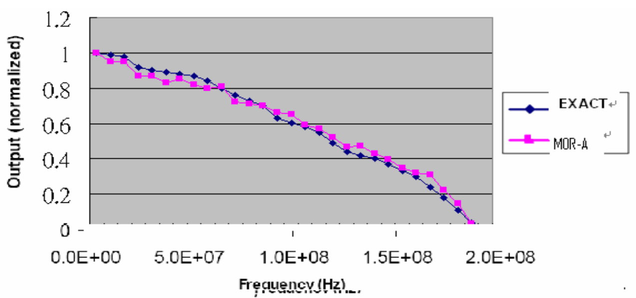

Figure-6. Model-A 3rd order HD comparison

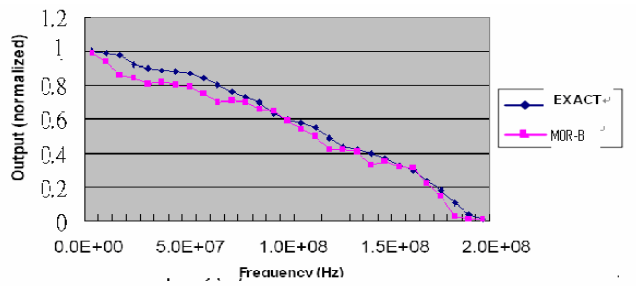

Figure-7 Model-B 3rd order HD comparison

Table-1. Error(%) of HD3 and IMD3 of model - A

Bench marks	Average HD3(db)		AverageIMD3(db)		Error (%)	
	SPICE	MOR-A	SPICE	MOR-A	HD3	IMD3
r1	1.23	1.21	-37.6	-32	1.6%	14.8%
r2	1.47	1.44	-41.2	-38	2.04%	7.7%
r3	0.94	0.89	-30	-22	5.3%	40%
r4	0.77	0.75	-21.4	-17	2.5%	20.5%

Table-2. Error (%) of HD3 and IMD3 of model - B

Bench marks	Average HD3 (db)		AverageIMD3 (db)		Error (%)	
	SPICE	MOR-B	SPICE	MOR-B	HD3	IMD3
r1	1.23	1.18	-37.6	-37	40.6%	1.5%
r2	1.47	1.37	-41.2	-40	6.4%	2.9%
r3	0.94	0.90	-30	-30	4.25%	0%
r4	0.77	0.73	-21.4	-17	5.1%	2.8%

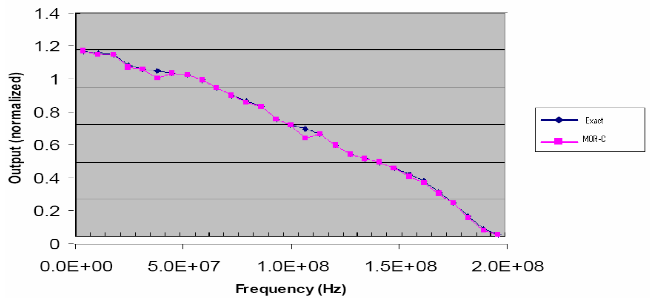

Figure-8. The 3rd order HD comparison for Model C with model order reduction

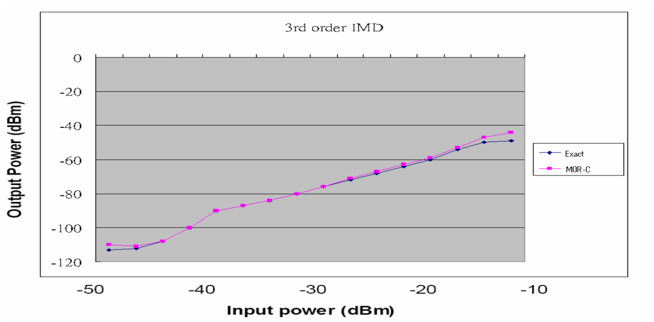

Figure-9. The 3rd order IMD comparison for Model C with model order reduction

Bench-marks	Average HD3(db)		Average P3(db)		Error %	
	SPICE	MOR-C	SPICE	MOR-C	HD3	IP3
r1	1.23	1.22	-37.6	-37.5	6	4
r2	1.47	1.46	-41.2	-41.1	6	4
r3	0.94	0.95	-30	-30.1	3	3
r4	0.77	0.76	-21.4	-21.5	2	4

Figure-10. Accuracy comparison of Model-C with model order reduction

VI. CONCLUSION

In this paper, we have presented an accurate, yet efficient frequency domain signal processing modeling in application to modern RF communication systems to describe the third order inter-modulation distortions and harmonic distortions based on Volterra series. Two basic model structures including a frequency shaping block and a resistive nonlinear gain block are analyzed algebraically and experimentally. We are also able to speed up the simulation time by using the model order reduction techniques on the proposed models.

Our analysis on the accuracy of the models shows that the proposed model - A gives relatively more accurate results on harmonic distortions (7%) than model - B (14%), and the proposed model - B gives relatively more accurate results of inter-modulation distortions (%16) than model - A (6%). The proposed compositional model achieves simulation results within 6% error on average in comparison with the results from Spectre simulator.

VII. REFERENCE

[1] Peter Feldmann and Roland W.Freund, “Efficient Linear Circuit Analysis by Pad ’ e Approximation via the Lanczos Process”, *IEEE Trans. on CAD*, vol. 14, no. 5, pp. 639-649, May ,1995.

[2] L.Miguel Silveira, Mattan Kamon and Jacob K.White, “Efficient Reduced-OrderModeling of Frequency-Dependent Coupling Inductances Associated with 3-d Interconnect Structures”, *Proc. of the IEEE/ACM DAC*, pp. 376-380, Jun. 1995.

[3] J. R. Phillips, “Projection Frameworks for Model Reduction of Weakly Nonlinear Systems”, *Proc. of IEEE/ACM DAC*, pp. 184-189, 2000.

[4] J.R.Phillips,“Automated Extraction of Nonlinear Circuit Macromodels”, *Proc. of IEEE Custom Itegrated Circuits Conference*,pp. 451-454, 2000.

[5] P.Dobrovolny, S.Donnay, M. Engels, and I. Bolsens,“Compact modeling of nonlinear distortion in analog communication circuits,” DATE’2000, pp. 350–354, 2000.

[6] Gielen, P.R.Kinget, W.Sansen, “High-frequency distortion analysis of analog integrated circuits,” *IEEE Trans. Circuits Syst.*, vol. 46, no. 3, pp. 335–345, March 1999.

[7] Y.S.Tang, and L.O.Chua, “Measuring Volterra kernels,” *IEEE Trans. Circuits Syst.,* CAS-30(8):571-577, August 1983.

[8] Feldmann, P.Freund, R.W, “Efficient linear circuit analysis by Pade approximation via theLanczos process,” IEEE Trans. on CAD, vol. 17, no. 8, pp. 645-654, May 1995