Model Order Reduction via Rational Transfer Function Fitting and Eigenmode Analysis

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Abstract: In this paper, we have proposed a novel model order reduction technique via rational transfer function fitting and eigenmode analysis considering residues. We define a constant as a key in the sorting algorithm as one of correlations in order to sort the order of eigenvalues. It is demonstrated that the accuracy via eigenmode analysis considering residues is improved. The proposed algorithm is a general method to match pole values with frequency domain poles for linear RC and RLC systems. Calculation of pole eigenvalues and eigen vectors can be done with more sophisticated analysis with the same level or smaller cost in the proposed algorithms in comparison to PRIMA. The experimental results show that our algorithm reduces up to 90% errors compared to the existing model order reduction algorithm, such as PRIMA, in wide frequency environment with the same number of poles in comparison.

Keywords: Model Order Reduction, PRIMA, Eigen-decomposition

1 INTRODUCTION

After entering deep sub-micron era, integrated circuits and systems are composed of a great number of linear (RLC) and non-linear, such as MOSFET or diode elements. The power grid (P/G) network may connect millions to tens-of-millions of transistors together. Therefore, it may cost a huge amount of computation time to analyze the whole network. In the past few years, model order reduction (MOR), as shown in Figure-1 has become a promising technology to accelerate this kind of simulation time for linear components.

Model reduction is undoubtedly one of the most useful aspects of system theory for simulation, because of its immediate relevance to model simplification. It combines mathematical modelling problems with computational complexity issues. However, mathematical models usually have some properties which are very important from the physical point of view such as conservativeness, dissipation, etc. As integrated circuits and systems continue to be designed with smaller size and faster operation, RLC interconnect effects have a more dominant impact on signal propagation than ever before. In addition, parasitic coupling effects and reduced power supply voltage levels make the interconnect modelling more important than ever before.

The existing model order reduction algorithms [7][8], such as PRIMA [1] using Krylov subspace projection matrix [5] to reduce order, is the most popular model order reduction in modern circuit simulation. However, PRIMA may cause severe accuracy problem in wide frequency environments.

Most of the moment matching [6][9] and projection framework model order reduction (like PRIMA) are only accurate around expansion point, but inaccurate on wide

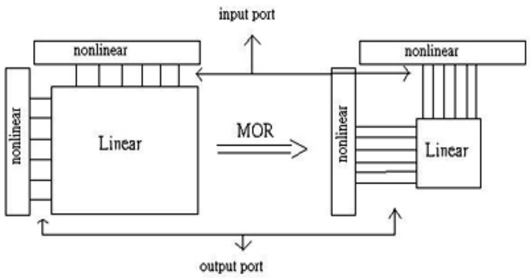

Figure 1 *Basic concept of MOR (model order reduction)*

frequency band. Point matching and eigenmode analysis model order reduction can keep accurate in approach point even in wide frequency band. However, it will be very inaccurate between those points. In this paper, we proposed a new model order reduction algorithms with advantages of both projection framework and eigenmode analysis model order reduction. The proposed method is able to be accurate in wide frequency, and it maintains accuracy between approach points [2][3] at the same time. The experimental results show that our proposed algorithm reduces up to 90% errors compared to the existing model order reduction algorithm, i.e., PRIMA in high frequency environment, with identical number of poles, say q poles.

It is understood that previous eigenmode analysis model order reduction does not consider residues, rational transfer function fittings and draping terms with small eigen values. It will be accurate only when residues are small. The main contribution in this paper is that the proposed algorithm provides an accurate yet efficient eigenmode analysis model order reduction. It matches 2q points while considering residues and rational transfer function fitting.

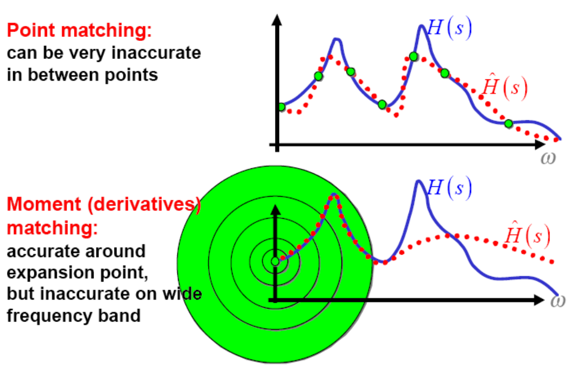

Figure 2 *Difference between Point matching and Moment matching*

2 PROPOSED ALGORITHM

2.1 Original Dynamical System

As shown in Figure. 2, voltage sources are connected to the ports to obtain the admittance matrix of a multiport [8]. The port, along with these sources, constitutes our time-domain modified nodal analysis (MNA) circuit equations. In Figure-3, *i* and *u* denote the port currents and voltages, respectively, and vectors *v* (*v1, v2, …*) are the MNA variables corresponding to the node voltages and the branch currents for voltage sources and inductors, respectively. The *C*, *G* matrices and *L* represent the conductance, susceptance matrices and the matrices containing the stamps for resistors, capacitors, and inductors, respectively. *L* consists of ones, minus ones, and zeros, which represent the current direction variables in KCL (Kirchhoff's Current Law) equations. The original port is assumed to compose of passive linear elements only with symmetric nonnegative definite matrices.

Using MNA [7] as shown in Figure-2, we can obtain a state matrix to describe the stamping circuits according KCL, KVL (Kirchhoff voltage law) and BCE (branch constitutive equations). In MNA form, we can get the original dynamical system function:

$$\frac{dx(t)}{dt} = A_r x(t) + b_r u(t)$$
$$y_r(t) = L_r^T x_r(t) \quad \ldots\ldots\ldots\ldots \quad (1)$$

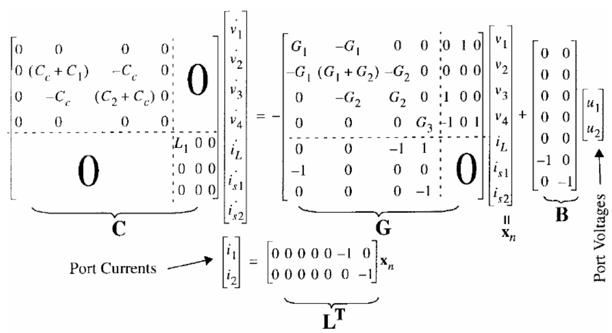

Figure 3 *Illustration of the formation of the MNA matrices for a two port RLC circuit*

With the transfer function:

$$H(s) = c_r^T (sI - A_r)^{-1} b_r \quad \ldots\ldots\ldots\ldots \quad (2)$$

H(s) transfer function is showed in frequency domain after Laplace transform (1). Thus, (2) can also be written as rational transfer function as follows [3]:

$$H(s) = \frac{b_1 + b_2 s + \ldots b_{n-1} s^{n-1}}{1 + a_2 s + \ldots a_n s^n} \quad \ldots\ldots\ldots\ldots \quad (3)$$

2.2 Eigenmode analysis

Mathematically we can solve all the ordinary differential equations (ODE) by using eigen-decomposition. In a linear system, MNA form from eq. (1), can be considered as an ODE:

$$\frac{dx(t)}{dt} = Ax(t) + bu(t)$$
$$x(0) = 0$$

Using eigen decomposition:

$$A=\begin{pmatrix}\vdots & \vdots & \vdots\\ E_1 & \cdots & E_N\\ \vdots & \vdots & \vdots\end{pmatrix}\begin{pmatrix}\lambda_1 & \dots & 0\\ 0 & \ddots & \vdots\\ 0 & \cdots & \lambda_N\end{pmatrix}\begin{pmatrix}\vdots & \vdots & \vdots\\ E_1 & \cdots & E_N\\ \vdots & \vdots & \vdots\end{pmatrix}^{-1} \quad \text{.........} \quad (4)$$

Changing variables $Ew(t)=x(t)$ into $w(t)=E^{-1}x(t)$

Substituting:

$$\frac{dEw(t)}{dt}=AEw(t)+bu(t) \quad , \; Ew(0)=0$$

And multiply by E^{-1}, we have

$$dw(t)/dt=E^{-1}AEw(t)+E^{-1}bu(t)$$

We can get new decoupled equations as below:

$$\frac{d}{dt}\begin{pmatrix}w_1\\ \vdots\\ w_N\end{pmatrix}=\begin{pmatrix}\lambda_1 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \lambda_N\end{pmatrix}\begin{pmatrix}w_1\\ \vdots\\ w_N\end{pmatrix}+\begin{pmatrix}(E^{-1}b)_1\\ \vdots\\ (E^{-1}b)_N\end{pmatrix}u(t)$$

And the output equation will be:

$$y(t)=L^T x(t)=L^T Ew(t)=(E^T L)^T w(t)=\tilde{C}^T w(t) \quad \text{.....} \; (5)$$

Solving decoupled equations:

$$w_i(t)=\int_0^t e^{\lambda_i(t-\tau)}\,\tilde{b}_i\,u(\tau)d\tau$$

And output equation:

$$y(t)=\sum_{i=1}^{N}\tilde{c}_i\,w_i(t)$$

Thus, we can get the reduced function

$$\begin{pmatrix}\dot{w}_1\\ \vdots\\ \dot{w}_N\end{pmatrix}=\begin{pmatrix}\lambda_1 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \lambda_q\end{pmatrix}\begin{pmatrix}w_1\\ \vdots\\ w_q\end{pmatrix}+\begin{pmatrix}\tilde{b}_1\\ \vdots\\ \tilde{b}_q\end{pmatrix}u(t) \quad \text{...................}(6)$$

With the reduced output equation:

$$y(t)=\sum_{i=1}^{q}\tilde{c}_i\,w_i(t)$$

Notice here, λ are sorted according to bubble sort algorithms [4]. Consider weights of bi in (3) and the values of λ which means:

$$\lambda_1>\lambda_2>\lambda_3>\lambda_4.....>\lambda_q$$

Physically, large eigen values (λ) represent dominate responses. We only keep terms with dominate response and ignore small ones to reduce the complexity of modified nodal analysis as our first step in the proposed algorithm.

Observe that certain modes are not affected by the input if $\tilde{b}_{k+1},............,\tilde{b}_N$ and $\tilde{c}_{k+1},............,\tilde{c}_N$ are small.

It also keeps least negative values (slowest modes):

$$w_i(t)=\int_0^t e^{\lambda_i(t-\tau)}\,\tilde{b}_i\,ud\tau=\frac{1}{\lambda_i}(\tilde{b}_i\,u-\tilde{b}_i\,ue^{\lambda_i t})$$

The value will be very small if $|\lambda_i|$ is large.

If $\tilde{b}_{k+1},............,\tilde{b}_N$ and $\tilde{c}_{k+1},............,\tilde{c}_N$ are large, it will cause inaccuracy under the assumption that we do not consider *bi* in the first step. This is the primary reason that we use bubble sort to consider the weight of bi in the first step of the proposed algorithm.

2.2 Improvement of the Proposed Algorithm

In our first step, the order of $\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_q$ are designed according to their values and certain weights of bi in (3). However, direct eigen decomposition will pay a huge calculation penalty if *G, C* matrix are large. Thus, we can get the reduced rational transfer function as:

$$H_r(s)=\frac{b_0^r+b_1^r s+.......+b_{q-1}^r s^{q-1}}{1+a_1^r s+.........+a_q^r s^q}$$

It can also be represented as:

$$H_r(s)=\frac{\tilde{c}_1\tilde{b}_1}{(s-\lambda_1)}+\frac{\tilde{c}_2\tilde{b}_2}{(s-\lambda_2)}+......\frac{\tilde{c}_N\tilde{b}_N}{(s-\lambda_N)}$$

The idea is that we define a constant as a key in the sorting algorithm, say $c_N b_N$ as one of correlations in order to sort the order in (6). To re-arrange the $\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_q$, we sort them according to the correlation of that constant and λ_i. From rational transfer function, we demonstrate a faster and lower calculation penalty by finding the dominated poles in the proposed algorithm. As a result, after keeping all the dominate poles, we cluster output functions in order to reduce the redundant poles.

3 EXPERIMENT RESULTS

All experimental data are measured in a Linux server with 1.9GHz CPU and 2GB memory. The results of the proposed algorithm are compared with PRIMA, as well as an exact value come from SPICE. Table - I summarizes the runtime results with RC benchmarks. RT indicates the computation run time in seconds. As we can see, the improvement on accuracy is significant, roughly 90% more accurate than PRIMA.

Node #	Frequency	PRIMA		Proposed	
		Error	RT(s)	Error	RT(s)
1208	0~10e3.5	17%	132	0.1%	342
4800	0~10e3.5	33%	203	1.4%	688
12000	0~10e3.5	47%	437	2.3%	1427
100000	0~10e3.5	49%	688	3.3%	1930

Table 1 *Runtime (RT) and accuracy testing result in proposed algorithm and PRIMA (RC mesh)*

Node #	Frequency	PRIMA		Proposed	
		Error	RT(s)	Error	RT(s)
1208	0~10e3.5	60%	214	0.4%	437
4800	0~10e3.5	77%	383	3.4%	597
12000	0~10e3.5	77%	497	4.8%	1734
100000	0~10e3.5	89%	644	6.1%	2840

Table 2 *Runtime (RT) and accuracy testing result in proposed algorithm and PRIMA (RLC mesh)*

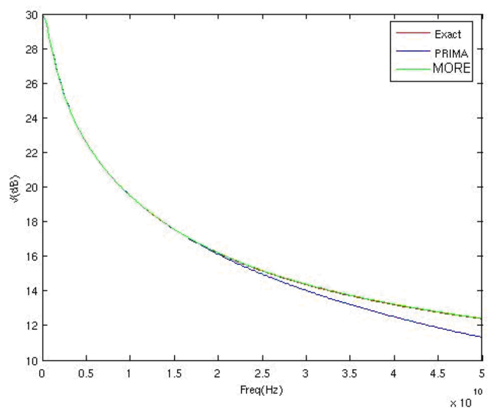

Figure 4 *Proposed algorithm vs PRIME with RC benchmark of 1208 nodes in wide frequency band*

Node: 4352, Edge: 5736, Port number: 1157
Matched moments:
1157 for the whole block for PRIMA (MIMO)
Original size: 4422x4422
Reduced size: 121x121 (Proposed), 1157x1157 (PRIMA)

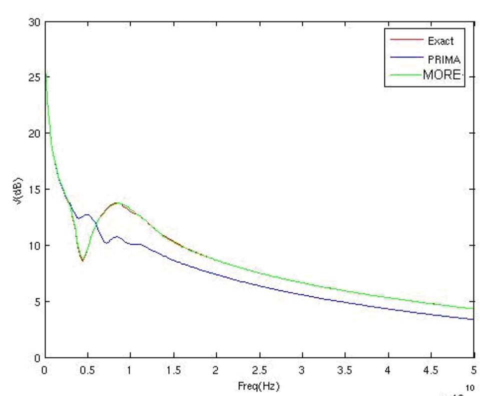

Figure 5 *Proposed algorithm vs PRIMA with benchmark of RLC circuits*

Node: 600, Edge: 776, Port number:110
Partition number: 4
Matched moments:
220 for the whole block for PRIMA (MIMO)
Original size: 988x988
Reduced size: 152x152 (Proposed), 220x220 (PRIMA)

Table II summarizes the runtime results, and the runtime comparisons with RLC benchmarks. The accuracy improvement is also significant, over 95%.

Note that the proposed algorithm can reduce up to 90% errors compared to the existing model order reduction algorithms, i.e. PRIMA in wide frequent environment and even more impressive as the circuit size increases.

In Figures 4 and 5, it shows the frequency domain responses for an RC mesh with 1208 nodes and RLC mesh 600 nodes. The proposed algorithm can match the original exactly, while PRIMA method shows a significant inaccuracy. The testing result shows the proposed algorithm is more accurate than PRIMA in wide frequency environments.

7 CONCLUSIONS

An extended model order reduction technique via rational transfer function fitting and eigen mode analysis order analysis of linear circuits with a large number of independent sources has been presented. This method can handle a much more wide frequency than existing algorithms. Experimental results show that the proposed method achieves a moderate accuracy improvement over circuit simulation with wide frequency variations and reduce up to 90% errors compared to the existing model order reduction.

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