# CGN 2420 Iterative Solutions and Optimization Using Excel Solver

Instructor: Professor Cora Martinez, PhD Department of Civil and Environmental Engineering Florida International University

# Objectives

- Find solutions, or "roots", of equations using a graph.
- Find roots of equations using several iterative solution methods:
  - "Guess and check" iteration.
  - Direct substitution.
  - Excel's Goal Seek, to solve for roots of equations.
  - Excel's Solver, to solve for roots of equations.
- Solve optimization problems using Excel's Solver

### Introduction

Some equations are easy to rearrange to solve for a variable, as an example, solve for the volume (V) of an ideal gas:

$$PV = nRT \Longrightarrow V = \frac{nRT}{P}$$

Try to solve for the porosity  $(\mathcal{E})$  in the following Eq:

$$\frac{\Delta P}{L} = \frac{150\overline{V_0}\mu}{\Phi_s^2 D_p^2} \cdot \frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{1.75\rho\overline{V_0}^2}{\Phi_s D_p} \cdot \frac{1-\varepsilon}{\varepsilon^3}$$

Iterative techniques can be used to solve a variety of complex equations.

## **Iterative Solutions**

- Nearly all iterative solution techniques require an initial guess to be provided by the user.
- Then the equation is solved using the guess value and a result is calculated.
- A test is performed to see if the solution is close enough to the correct answer.
- If it is not, a new guess value is used; the repeating process is called *iteration*.

## **Standard Forms**

The equation to be solved should be written into a standard form. Consider the equation

$$x^3 + 12 = 17x$$

Form	Example	Convenient for
(1) Set Equation equal to Zero	$x^{3}-17x+12=0$	•Plot Method •Excel's Solver
(2) Get an "x" by itself on the left side	$x=(x^3+12)/(17)$	<ul><li>Direct-substitution</li><li>Method</li><li>In cell Iteration</li></ul>



# Using a Plot to Search for Roots

#### Procedure:

- Set equation equal to zero.
- To search for roots, we will try various values of x and solve for f(x) (when a change of sign is found then is known a root is in between).
- Graph f(x) vs. x.

Number of roots must be equal to degree of equation.

### Using a Plot to Search for Roots (Cont.)



## **Guess and Check Iteration**

One of the easiest ways to find the root is simply to create a worksheet with a place to enter guess values (x) and a formula that evaluates f(x).

- F(x) can be inserted in form 1 or form 2.
- A tolerance value is used to judge whether the value is close enough to zero (form1) or to the guessed value (form 2).

#### Guess and Check Iteration (Cont.)

Only one root can be found at a time.

		· (° - 🖸	Ŧ								E
	Home	Insert	Page Layout	Formulas	Data	Review	View	Develo	oper		
Pas	National Copy Stell	at Painter	Calibri -	• 11 • .	A A	= = <b>●</b> → ■ w E = = # # # ⊠ M		ar M Barner M	/rap Text lerge & Center +		General \$ - १
	Clipboard	5	Fo	ont	5		Alignm	ient		5	N
	B32	- (	f <sub>x</sub>	=A32^3-	17*A32+	+12					
	А	В	С	D	E	F		G	Н		1
22	Simple Gue	ess and Che	eck Iteration								
23											
24	Guess	f(x)									
25	0.8	-1.088									
26	0.7	0.443									
27	0.75	-0.32813									
28	0.72	0.133248									
29	0.73	-0.02098									
30	0.725	0.056078									
31	0.727	0.025241									
32	0.728	0.009828									
33										1	

# **Direct Substitution Technique**

- This method uses the previous computed value as the next guess value.
- The equation is entered using standard form
   2.
- Simple method but some roots cannot be found using it.

First guess has to be entered by hand.

### Direct Substitution Technique (Cont.)

	1	- (4 - 🖸 )	Ŧ					
	Home	Insert P	age Layout	Formulas	Data	Review Vie	ew Develo	oper
Pa	Cut La Copy ste , ✓ Form	Ca at Painter	libri I <u>U</u> -	• 11 • 1		= <mark>=</mark> »		rap Text erge & Center
6	Clipboard	G	Fo	ont	5	4	Alignment	
	B38	• (*	$f_{x}$	=(A38^3+	-12)/17			
	А	В	С	D	E	F	G	Н
34								
35	Direct Sub	stitution M	ethod					
36								
37	Xguess	Xcomputed						
38	0.8	0.736						
39	0.736	0.7293346						
40	0.729335	0.7287032						
41	0.728703	0.728644						
42	0.728644	0.7286384						
43	0.728638	0.7286379						
44	0.728638	0.7286379						
45	0.728638	0.7286378						
46	0.728638	0.7286378						
47	0.728638	0.7286378						
48								
49								

# Using Goal Seek in Excel

This feature allows you to solve problems backwards: to find the input values needed to generate the answers you want.

Procedure:

- Express equation using form 1
- Set a cell to hold the guess value
- Set a cell containing the formula (form 1)
- Open the Goal Seek dialog, use ribbon options: Data/Data Tools/What-if Analysis/Goal Seek

# Using Goal Seek in Excel (Cont.)

- Set cell that contains the formula (B53) to zero.
- The cell that will be changed is the one set for the guess value.

		· 🗘 · 🖄	<b>,</b>											E
	Home	Insert	Page Layout	Forn	nulas	Data	Re	eview	Vie	w I	Developer			
Fro	om From ess Web	From From Text Sou Get External [	o Other Existi rces * Connec Data	ng tions	Refres All *	D Connect	onnect operti lit Link ions	tions ies cs	Az↓ Z↓	AZA Sort	Filter Sort & Filt	Ҡ Clear 🚡 Rear 🖤 Adva er	r oply anced	Text t Colum
	B53	<b>•</b> (	fx f	=B5	2^3-1	7*B52+	-12							
	А	В	С	D		E		F		G		Н		I
49														
50	Using Goa	l Seek In E	xcel	Goa	l Seek				?	$\mathbf{X}$				
51				Set	cell:	e	53		6					
52	Guess:		_	To y	alue					-				
53	f(x):	1	.2	Du el	onue.	colly d	0450		6					
54				Буц	nanyiny	cen: ‡	5932		LE	<u> </u>				
55						ОК		Ca	ncel					
57														
58														
59														
60														
61														
62														

Click OK.

# Using Goal Seek in Excel (Cont.)

	0 2 7	• (°" + [ <b>g</b> ]) :	Ŧ								
	Home	Insert P	age Layout	Form	ulas	Data	Reviev	/ Vi	ew C	)evelope	r
Fro	m From ess Web	From From O Text Source	ther Exis	sting	Refres All •	Der Cor Pro © Edit	nnections perties t Links	A Z↓ Z↓	A Z Z A Sort	Filter	K Clear Reap Adva
		Get External Dat	a			Connectio	ons		;	Sort & Fi	lter
	B53		$f_s$	= =B52	2^3-17	7*B52+3	12				
	А	В	С	D		E		F	G		Н
49											
50	Using Goa	l Seek In Exc	el	Goal S	eek Si	atus			<u>?</u>		
51				Goal Se	eking w	ith Cell B5	53	St	ep		
52	Guess:	0.7286356		found a	solution	٦.					
53	f(x):	3.396E-05		Target	/alue:	0		Pat	JSE		
54				Current	value:	3.3956E	-05				
55					ſ	OK		Car	ncel		
56					L	OK		Cai	icor	J	
57											
58											
59											
60											

- Clicking OK causes Excel to change the cell values until a solution is met.
- Goal Seek leaves the root in the guess cell.
- In other to find the other roots, the initial guess must be changed.

# Introduction to Excel's Solver

- The Goal Seek command is handy for problems that involve an exact target value that depends on a single unknown value.
- For problems that are more complex, you should use the Solver add-in.
- The Solver can handle problems that involve many variable cells and can help you find combinations of variables that maximize or minimize a target cell.
- It also specifies one or more constraints conditions that must be met for the solution to be valid.

## **Activating Solver**

To use solver in Office 2007, Click the Office button, then click the Excel Options button at the bottom – You'll see a screen like this:

Popular Formulas	Change the most popular options in Excel.	
Proofing	Top options for working with Excel	
Save Advanced Customize Add-Ins	✓       Show Mini Toolbar on selection ③         ✓       Enable Live Preview ④         □       Show Developer tab in the Ribbon ③         Color scheme:       Blue ▼         ScreenTip style:       Show feature descriptions in ScreenTips ▼	
Trust Center	Create lists for use in sorts and fill sequences: Edit Custom Lists	
Resources	When creating new workbooks	
	Font size: 11  Default yiew for new sheets: Normal View Include this many sheets: 3	
	Personalize your copy of Microsoft Office	
	User name: Kristian Friel Choose the languages you want to use with Microsoft Office: Language Settings	
	ОК	Canc

# Activating Solver (Cont.)

Click Add-Ins, In the Manage box, choose Excel Add-ins, select Solver Add-in, Click Go...

Popular Formulas	View and manage Microsoft Office add	-ins.	
Proofing	Add-ins		
Save	Name	Location	Туре
Advanced	Active Application Add-ins	C. 199714.1. 1977. 19971497	CONTRACTO
Customize	DYMO LabelWriter Addin	C:\IPDFMaker\Office\PDFMOfficeAddin.dll C:\iles\DYMO Label\DymoOfficeAddIn.dll	COM Add-in
Customize	PaperWise SendTo Addin for Microsoft Excel 2000	C:\ise\PaperWise Suite V6\PwOfficeBar.dll	COM Add-in
Add-Ins	Solver Add-in	C:\Office12\Library\SOLVER\SOLVER.XLAM	Excel Add-in
Trust Center	Inactive Application Add-ins		
Bacourcas	Analysis ToolPak	C:\\Office12\Library\Analysis\ANALYS32.XLL	Excel Add-in
Resources	Analysis ToolPak - VBA	C:\fice12\Library\Analysis\ATPVBAEN.XLAM	Excel Add-in
	Conditional Sum Wizard	C:\oft Office\Office12\Library\SUMIF.XLAM	Excel Add-in
	Custom XML Data	C:\\Microsoft Office\Office12\OFFRHD.DLL	Document Inspecto
	Date (Smart tag lists)	C:\s\microsoft shared\Smart Tag\MOFL.DLL	Smart lag
	Euro Currency Tools	C:\ffice\Office12\Library\EUROTOOL.XLAM	Excel Add-in
	Financial Symbol (Smart tag lists)	C:\s\microsoft shared\Smart Tag\MOFL.DLL	Smart lag
	Headers and Footers	C:\\Microsoft Office\Office12\OFFRHD.DLL	Document Inspecto
	Hidden Rows and Columns	C:\\Microsoft Office\Office12\OFFRHD.DLL	Document Inspecto
	Hidden Worksheets	C:\\Microsoft Office\Office12\OFFRHD.DLL	Document Inspecto
	Internet Assistant VBA	C:\soft Office\Office12\Library\HTML.XLAM	Excel Add-in
	Invisible Content	C:\\Microsoft Office\Office12\OFFRHD.DLL	Document Inspecto
	Lookup Wizard	C:\t Office\Office12\Library\LOOKUP.XLAM	Excel Add-in
	Person Name (Outlook e-mail recipients)	C:\microsoft shared\Smart Tag\FNAME.DLL	Smart Tag
	Document Related Add-ins		
	Add-in: Acrobat PDFMaker Office COM Ad	din	
	Publisher: Adobe Systems, Incorporated		
	Location: C:\Program Files\Adobe\Acrobat 8	.0\PDFMaker\Office\PDFMOfficeAddin.dll	
	Description: Acrobat PDFMaker Office COM Ad	ain	
	Manage: Excel Add-ins		

# Activating Solver (Cont.)

Check the Solver Add-in box, and click OK.



## Solver Components

Solver has three core components:

Target cell (objective function).
 Changing cell (design variables).
 Constraints.

The target cell represents the objective or goal that we want to achieve.



# Solver Components (Cont.)

- The changing cell represent the range of variables from which the solver selects the specific number(s) that satisfies the optimization.
- The constraints are restrictions that we impose when constructing the function to be solved.



# What options can we choose?

Solver has several options you can play with to optimize your results:

Max <u>T</u> ime:	100 seconds	ОК
Iterations:	100	Cancel
Precision:	0.000001	Load Model
Tol <u>e</u> rance:	5 %	Save Model
Con <u>v</u> ergence:	0.0001	Help
Assume Line	ar Model 📃 📃	Ise Automatic Scaling
Assume Non	-Negative 📃 S	how Iteration <u>R</u> esults
Estimates	Derivatives	Search
Tangent	Eorward	Newton
and the second second	0	

# Finding Roots With Solver

 Solver requires that the equation be written in a variation of standard form 1.

Procedure:

- Express equation using form 1.
- Set a cell to hold the guess value.
- Set a cell containing the formula (form 1).
- Start the Solver using menu options: Data/Analysis/[Solver]
- The solver parameter dialog will be displayed.



### Finding Roots with Solver (Cont.)

- The target cell is the cell containing the formula.
- Set target cell equal to Zero. By doing this Excel will continue guessing until the formula is equal to zero.
- Set the value of the field "By Changing Cell" equal to the cell that holds the guess value.



#### Finding Roots with Solver (Cont.)



#### Finding Roots with Solver (Cont.)

- Once the required information has been set, click on the solve button to iterate for a solution.
- Here the Solver Results Dialog indicates that a solution was found.
- To search for other roots, set a new guess value and run the Solver again.

		- (2 - 13)	Ŧ					
	Home	Insert P	age Layout	Formulas	Data F	Review	View D	Developer
Frc	om From ess Web	From From C Text Source	other Exist es * Conne	ting ctions All *	Denne Proper © Edit Lin	ctions ties nks	Z Z A Sort	Filter
	566	Get External Da	ta		Connections			Sort & Filter
	E66	• (*	Jx		-	-		0
55	A	В	C	D	E	F		G
56	Using the	Solver						
57	o sing the							
58	Guess:	0.7286379	←	Cell to be ch	nanged			
59	f(x):	-6.26E-07	←	Target Cell				
60								
61								
62								
63								
64	Solver Resu	ilts					×	
65 66	Solver found conditions are	a solution. All ( e satisfied.	constraints ar	nd optimality	Repo	rts		
67	() Keep So	olver Solution			Answ Sensi	er itivity	~	
69	O Restore	Original Values	3		Limits	3	~	
70			- Incol	Caus Cau				
71	UK			<u>Save Scer</u>			anh an	
72								
73								

The Solver can also be used for optimization problems. As a simple example, consider:

$$y = 10 + 8x - x^2$$

This equation has a maximum value of 26 at x=4



- Set the target cell as the cell containing the formula.
- Set the target cell equal to "Max".
- Set changing cell as the cell that holds the guess value.

Cli	ck	SO	lve.

	) 🛛 🤊	• (° • 🖸 )	Ŧ							E		
	Home	Insert P	age Layout	Formulas	Data	Review	View	Developer				
Fro	From       Existing       Connections       All ~       Sort       Filter       Advanced       Text       Column         Get       External Data       Connections       Connections       Connections       Sort & Filter       Advanced       Column         B85       Connections       Filter       EBS8A3-17*B58+12       Filter       Sort & Filter       Filter											
	B85	<b>-</b> (0	$f_x$	=B58^3-1	7*B58+	12						
	А	В	С	D	E		F	G	Н	I.		
83	Optimizat	ion Using th	e Solver									
84												
85	Guess:	2										
86	Result:	22										
87												
88	Solver P	arameters										
89	Cot Torra	at Cally	tator I	sel				Calua	-			
90	Equal To		\$B\$80	🚬 🔾 Valua et				Solve	-			
92	- By Chan	 aina Cells:			·			Close	-			
93	\$B\$85					Guess						
94	Subject	to the Constrain	te :									
95		o the constitution				6 d d		options	1			
96						Auu						
97						<u>C</u> hange		Reset All				
98						Delete		70061 MII				
99								Help				
100									3			
101												

 Here the Solver Results Dialog indicates that a solution was found.

		- (2 - 12);	Ŧ							
	Home	Insert Pa	age Layout	Form	nulas	Data Re	view	View	Develop	er
Fro	om From ess Web	From From O Text Source	ther Exist	ting	Refresh All *	Dennect	ions es ːs	Ž↓ <u>A</u> Z↓ Sor	rt Filter	K K
	Get External Data Connections Sort 8									
-	880		Jx	=10	+8*B85	D-B85/2	-			
<u> </u>	A	B	Calvan	-	D	E	F	•	G	
83 84	Optimizati	ion Using the	Solver							
85	Guess:	4								
86	Result:	26								
87										
88										
89										
90	Solver Resu	ılts							×	
92 93	Solver found conditions are	a solution. All c e satisfied.	onstraints ar	ıd optim	nality	Reports	3			
94 95	<ul> <li>Keep So</li> <li>Restore</li> </ul>	olver Solution				Answei Sensiti Limits	r vity	~		
90 97 98	ОК	Ca	incel	<u>S</u>	ave Scen	ario	E	<u>l</u> elp		

Adding Constrains:

- If you want to find the maximum value of y for  $x \le 3$ , a constrain must be added.
- To include a constrain:
  - Click the Add Button at the right of the constrains box.
  - Add constrain.



#### Adding Constrains:

	А	В	С	D	E	F	G	0010010010
83	Optimizati	on Using th	e Solver					
84								
85	Guess:	2						
86	Result:	22						
87								
Sol	ver Parame	ters					X	
Set	: Target Cell:	\$B\$86				<u>S</u> ol	lve	
Equ	ual To: 🛛 🤅	) <u>M</u> ax 🔿 I	Mi <u>n Oy</u>	/alue of: 0		Clo		
By	Changing Cel	ls:						
\$	B\$85				<u>G</u> uess			
S	ibject to the C	onstraints:				Opti	ons	
					<u>A</u> dd	2		
				(	<u>C</u> hange		t All	-
				<b>a</b>	Delete			
L								
100								



Add Constraint			X
Cell Reference :		Constraint:	
\$B\$85	🧾 <= 🔽	3	
ОК Са	ancel	Add Help	

	А	В	С	D	E	F	G	
83	Optimizati	on Using th	e Solver					
84								
85	Guess:	3						
86	Result:	25						
Sol	ver Results					Þ	<	
Solver found a solution. All constraints and optimality conditions are satisfied.       Reports            • Keep Solver Solution         • Restore Original Values         • Cancel         • Save Scenario         • Help         • H								
95								
96								
07								

#### Solving for Multiple Values:

The function: f(x, y) = sin(x).cos(y)Has a maximum at x=1.5708 and y=0 in the region ( $-1 \le x \le 2$ ,  $-1 \le y \le 2$ ).



Solving for Multiple Values:

• The solver will change multiple cells to try to find the requested result.

#### Procedure:

- Enter guesses for x and y, and the equation to be solved.
- Enter constrains.
- Click Solve.



#### Solving for Multiple Values:

	А	В	С	D	E	F	G	Н		
128	Optimizati	ion Using th	e Solver							
129										
130		x	У							
131	Guess:	0	0							
132	Result:	0								
133								107		
134	Solver Pa	arameters						1		
135						C		-		
136	Set Targe	Set Target Cell: Solve								
137	Equal To:	) <u>M</u> ax	() Mi <u>n</u>	() <u>V</u> alue of:	U	(	Close			
138	by Chang	ing cells:		(						
139	\$B\$131:	\$C\$131		E		ess				
140	-S <u>u</u> bject t	o the Constraint	s:				Options			
141	\$B\$131	<= 2			<u>A</u>	bt				
142	\$B\$131 \$C\$131	>=-1 <=2			Cha	nge				
143	\$C\$131	>= -1					<u>R</u> eset All			
144						ete	Help			
145										
146										
147										



4	А	В	С	D	E	F	G			
128	Optimizati	ion Using th	e Solver							
129										
130		х	у							
131	Guess:	1.5707963	-3E-08							
132	Result:	1								
133										
134										
135										
13 s	Solver Results									
13	Solver found a conditions are	a solution. All c satisfied.	onstraints an	d optimality	Reports					
13 14 14	Keep So Restore	lver Solution) Original Values			Answer Sensitiv Limits	ity				
14 14	ОК	Ca	ncel	<u>S</u> ave Scena	ario	<u>H</u> elp				
144 145										