

# CGN 2420

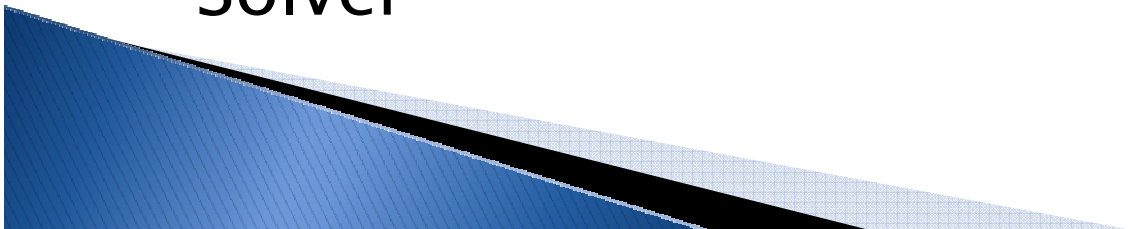
## Iterative Solutions and Optimization Using Excel Solver

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Florida International University



# Objectives

- ▶ Find solutions, or “roots”, of equations using a graph.
- ▶ Find roots of equations using several iterative solution methods:
  - “Guess and check” iteration.
  - Direct substitution.
  - Excel’s Goal Seek, to solve for roots of equations.
  - Excel’s Solver, to solve for roots of equations.
- ▶ Solve optimization problems using Excel’s Solver



# Introduction

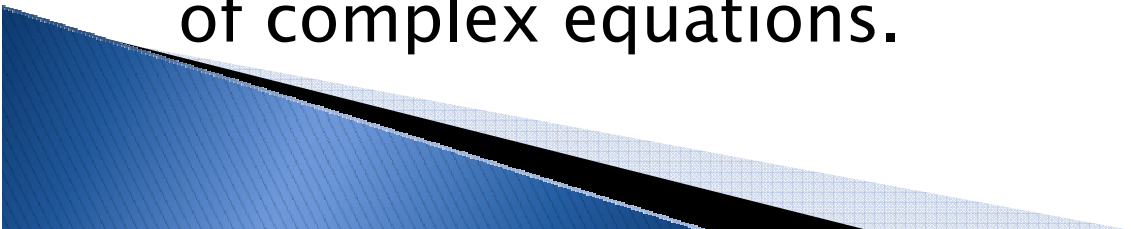
Some equations are easy to rearrange to solve for a variable, as an example, solve for the volume ( $V$ ) of an ideal gas:

$$PV = nRT \Rightarrow V = \frac{nRT}{P}$$

Try to solve for the porosity ( $\epsilon$ ) in the following Eq:

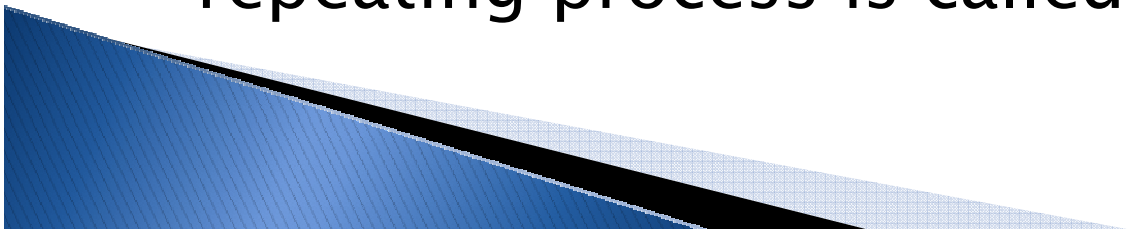
$$\frac{\Delta P}{L} = \frac{150\bar{V}_0\mu}{\Phi_s^2 D_p^2} \cdot \frac{(1-\epsilon)^2}{\epsilon^3} + \frac{1.75\rho\bar{V}_0^2}{\Phi_s D_p} \cdot \frac{1-\epsilon}{\epsilon^3}$$

Iterative techniques can be used to solve a variety of complex equations.



# Iterative Solutions

- ▶ Nearly all iterative solution techniques require an initial guess to be provided by the user.
- ▶ Then the equation is solved using the guess value and a result is calculated.
- ▶ A test is performed to see if the solution is close enough to the correct answer.
- ▶ If it is not, a new guess value is used; the repeating process is called *iteration*.

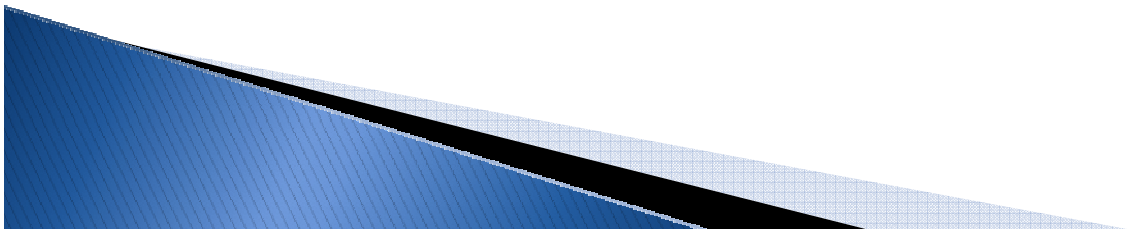


# Standard Forms

- ▶ The equation to be solved should be written into a standard form. Consider the equation

$$x^3 + 12 = 17x$$

Form	Example	Convenient for..
(1) Set Equation equal to Zero	$x^3 - 17x + 12 = 0$	<ul style="list-style-type: none"><li>•Plot Method</li><li>•Excel's Solver</li></ul>
(2) Get an "x" by itself on the left side	$x = (x^3 + 12) / (17)$	<ul style="list-style-type: none"><li>•Direct-substitution Method</li><li>•In cell Iteration</li></ul>

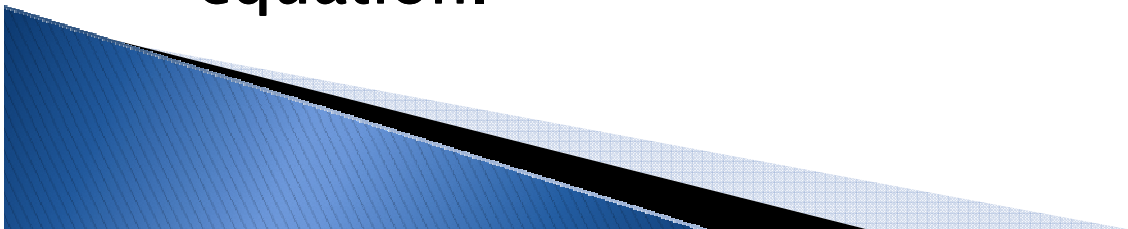


# Using a Plot to Search for Roots

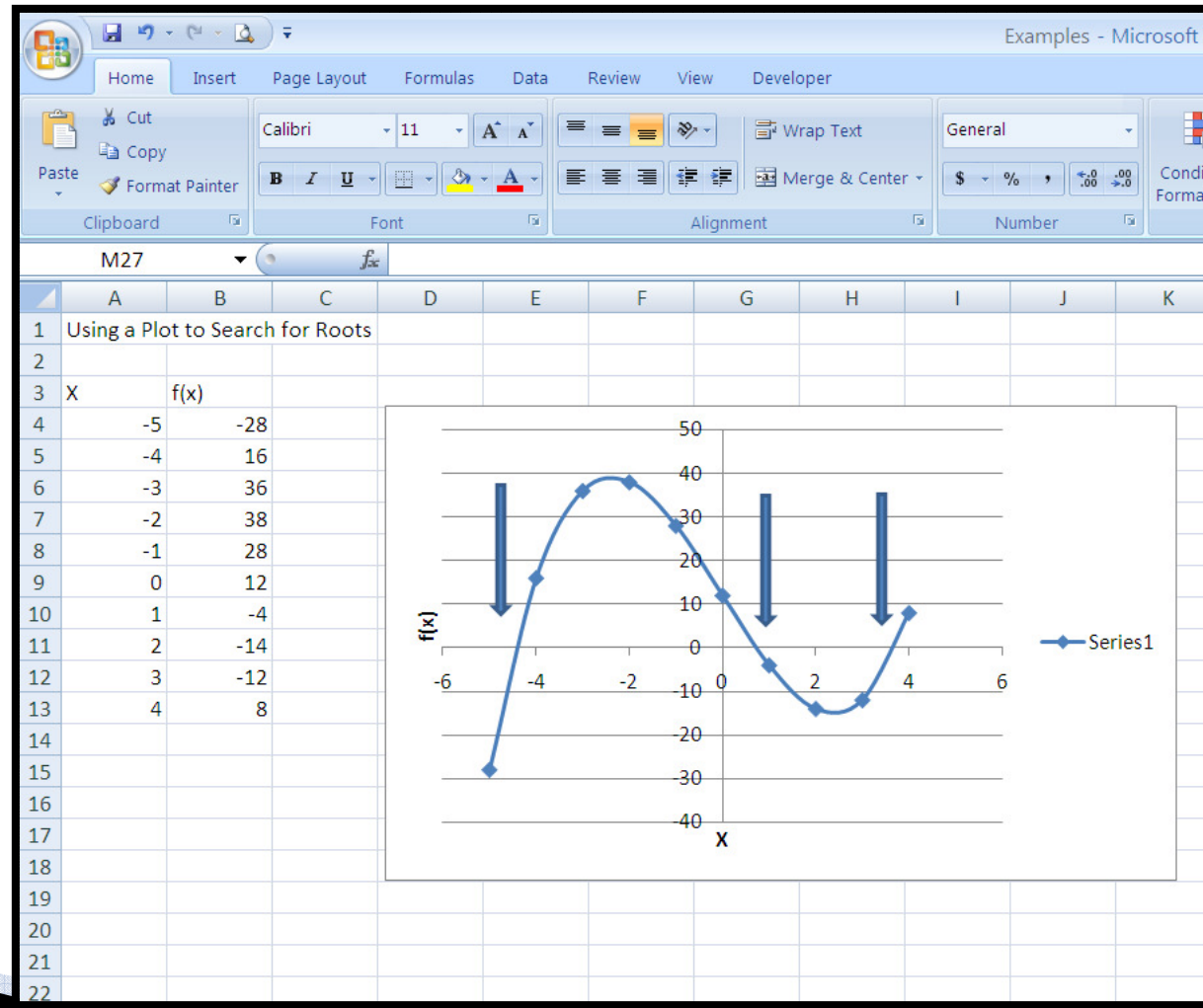
## Procedure:

- ▶ Set equation equal to zero.
- ▶ To search for roots, we will try various values of  $x$  and solve for  $f(x)$  (when a change of sign is found then is known a root is in between).
- ▶ Graph  $f(x)$  vs.  $x$ .

Number of roots must be equal to degree of equation.



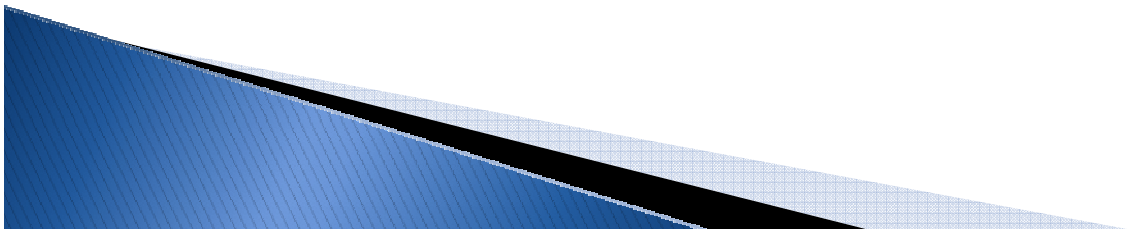
# Using a Plot to Search for Roots (Cont.)



# Guess and Check Iteration

One of the easiest ways to find the root is simply to create a worksheet with a place to enter guess values ( $x$ ) and a formula that evaluates  $f(x)$ .

- ▶  $F(x)$  can be inserted in form 1 or form 2.
- ▶ A tolerance value is used to judge whether the value is close enough to zero (form 1) or to the guessed value (form 2).

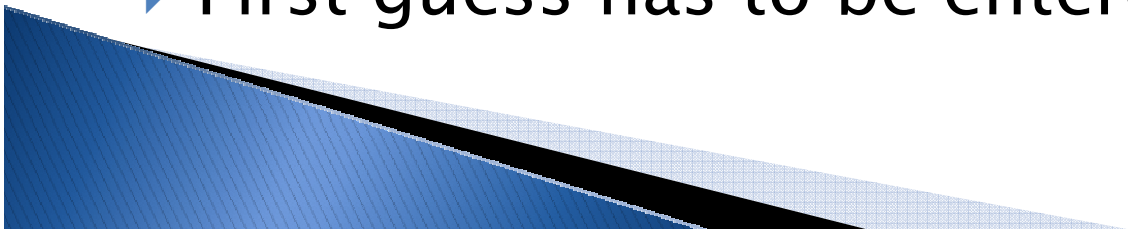






# Direct Substitution Technique

- ▶ This method uses the previous computed value as the next guess value.
- ▶ The equation is entered using standard form 2.
- ▶ Simple method but some roots cannot be found using it.
- ▶ First guess has to be entered by hand.



# Direct Substitution Technique (Cont.)

The screenshot shows the Microsoft Excel interface with the following data in the spreadsheet:

	A	B	C	D	E	F	G	H
34								
35	<b>Direct Substitution Method</b>							
36								
37	Xguess	Xcomputed						
38	0.8	0.736						
39	0.736	0.7293346						
40	0.729335	0.7287032						
41	0.728703	0.728644						
42	0.728644	0.7286384						
43	0.728638	0.7286379						
44	0.728638	0.7286379						
45	0.728638	0.7286378						
46	0.728638	0.7286378						
47	0.728638	0.7286378						
48								
49								

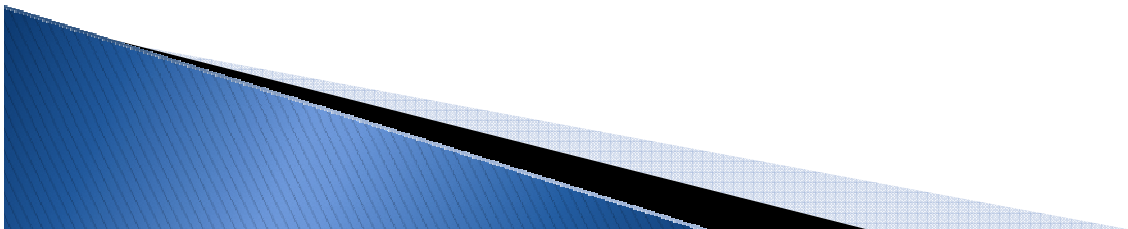
The formula bar at the top shows the equation:  $x^3 + 12 = 17$ .

# Using Goal Seek in Excel

This feature allows you to solve problems backwards: to find the input values needed to generate the answers you want.

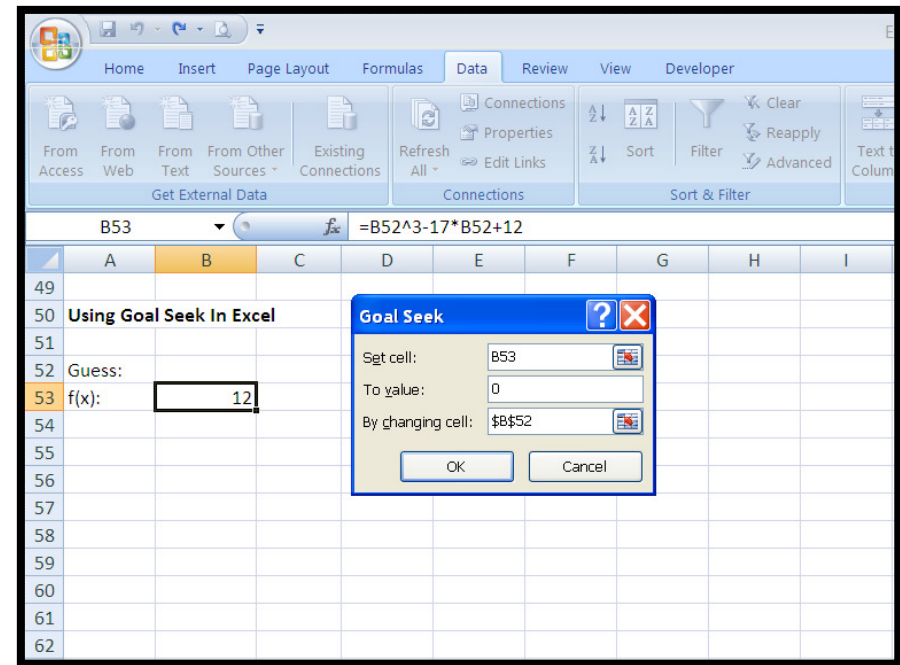
## Procedure:

- Express equation using form 1
- Set a cell to hold the guess value
- Set a cell containing the formula (form 1)
- Open the Goal Seek dialog, use ribbon options:  
Data/Data Tools/What-if Analysis/Goal Seek

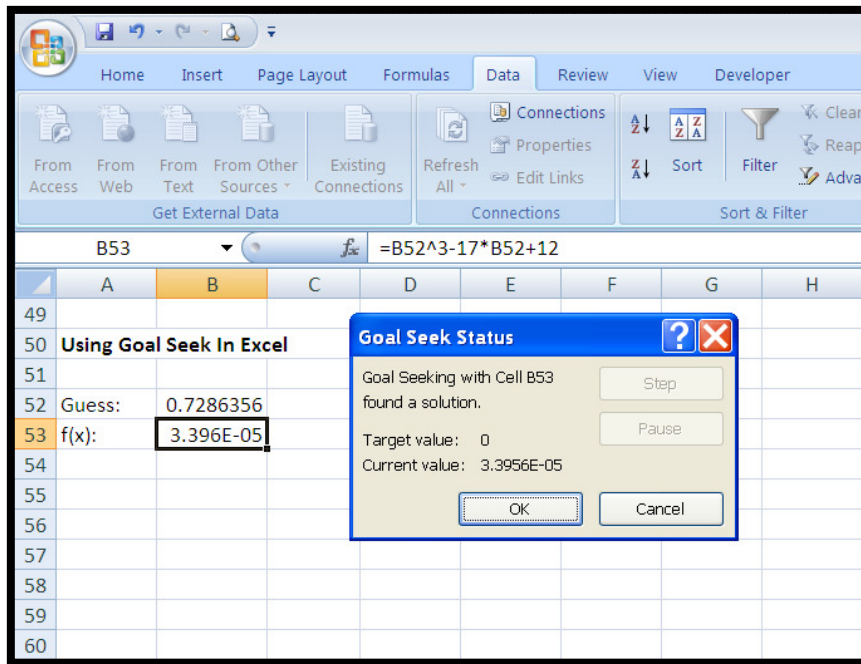


# Using Goal Seek in Excel (Cont.)

- ▶ Set cell that contains the formula (B53) to zero.
- ▶ The cell that will be changed is the one set for the guess value.
- ▶ Click OK.



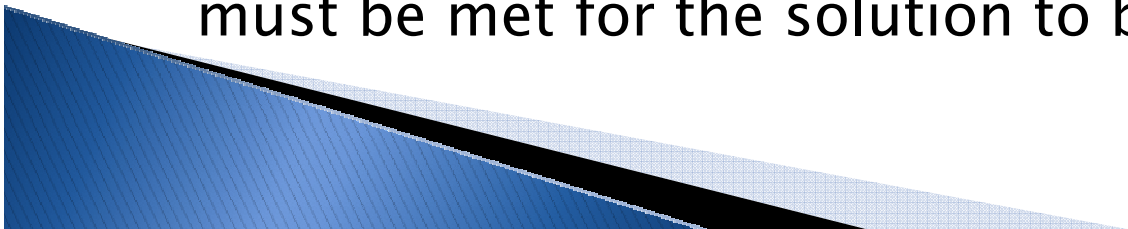
# Using Goal Seek in Excel (Cont.)



- ▶ Clicking OK causes Excel to change the cell values until a solution is met.
- ▶ Goal Seek leaves the root in the guess cell.
- ▶ In order to find the other roots, the initial guess must be changed.

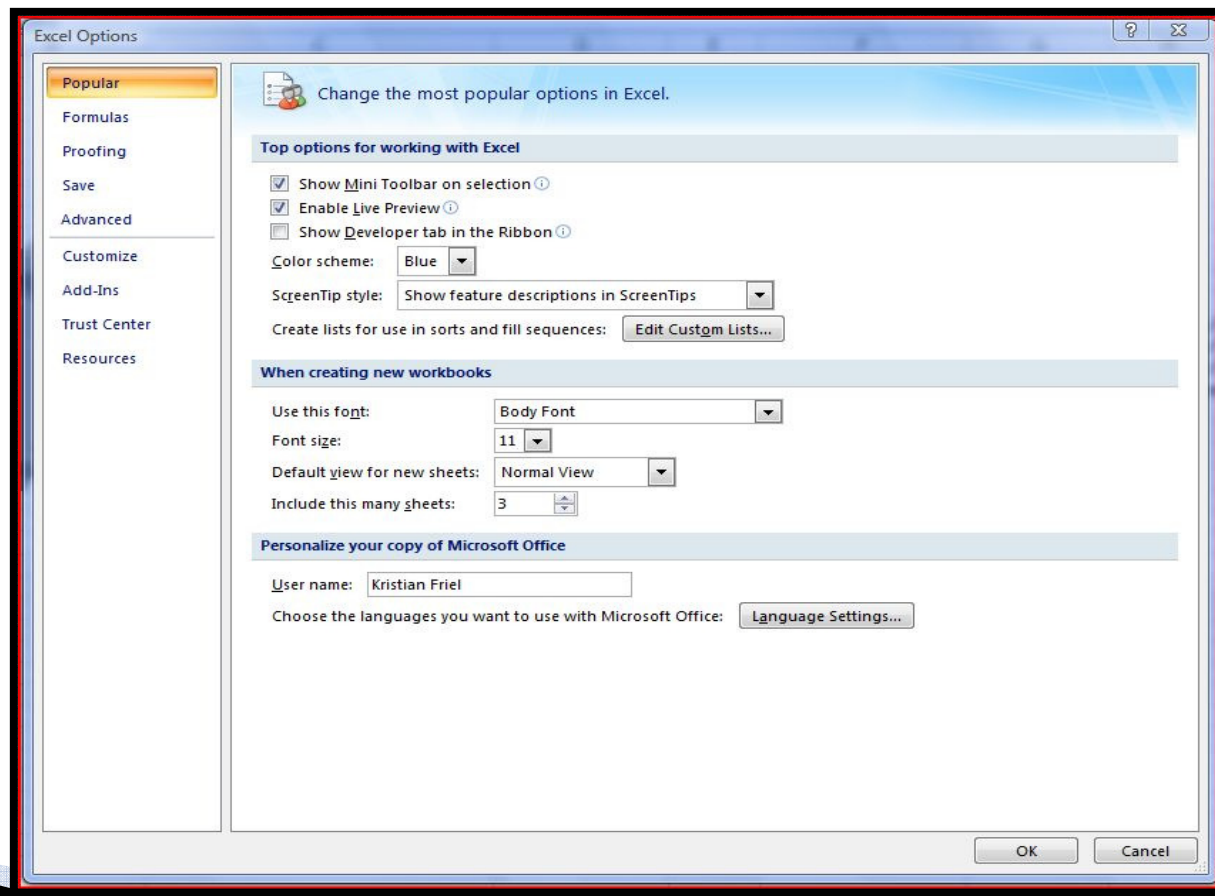
# Introduction to Excel's Solver

- ▶ The Goal Seek command is handy for problems that involve an exact target value that depends on a single unknown value.
- ▶ For problems that are more complex, you should use the Solver add-in.
- ▶ The Solver can handle problems that involve many variable cells and can help you find combinations of variables that maximize or minimize a target cell.
- ▶ It also specifies one or more constraints conditions that must be met for the solution to be valid.



# Activating Solver

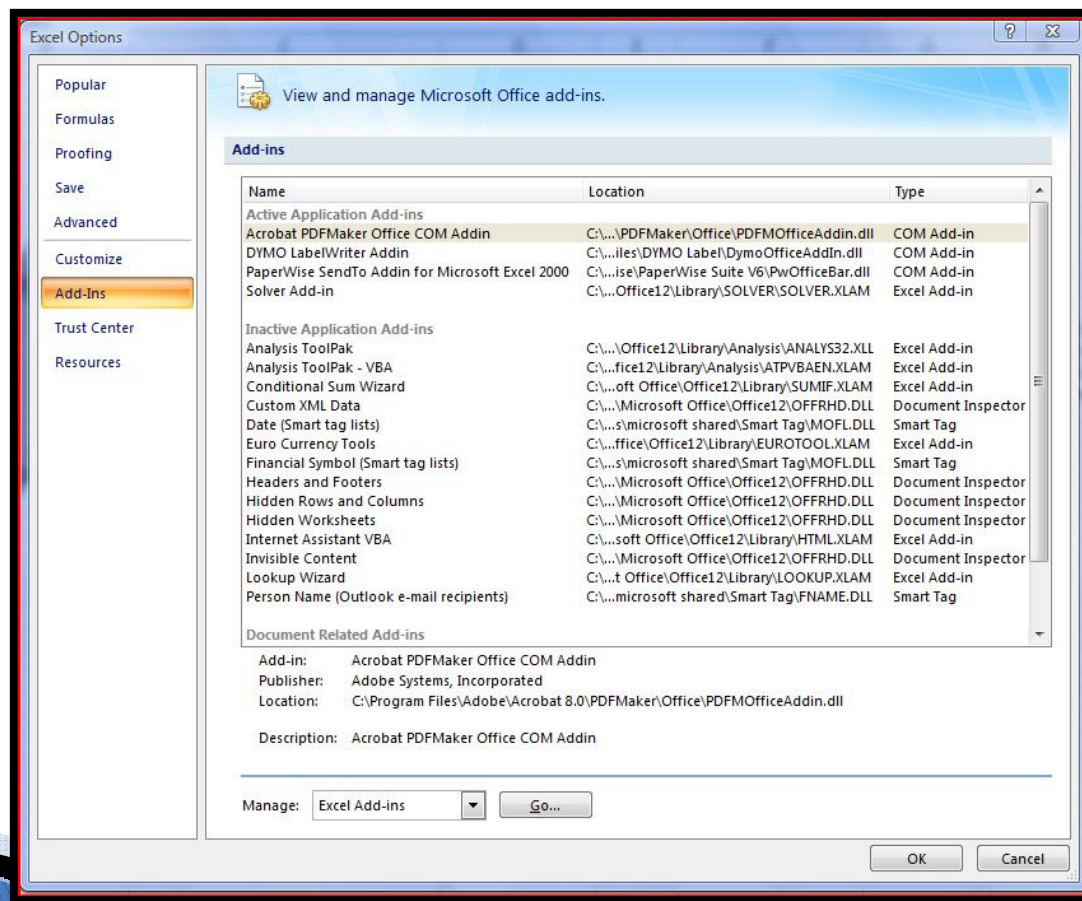
To use solver in Office 2007, Click the Office button, then click the Excel Options button at the bottom – You'll see a screen like this:





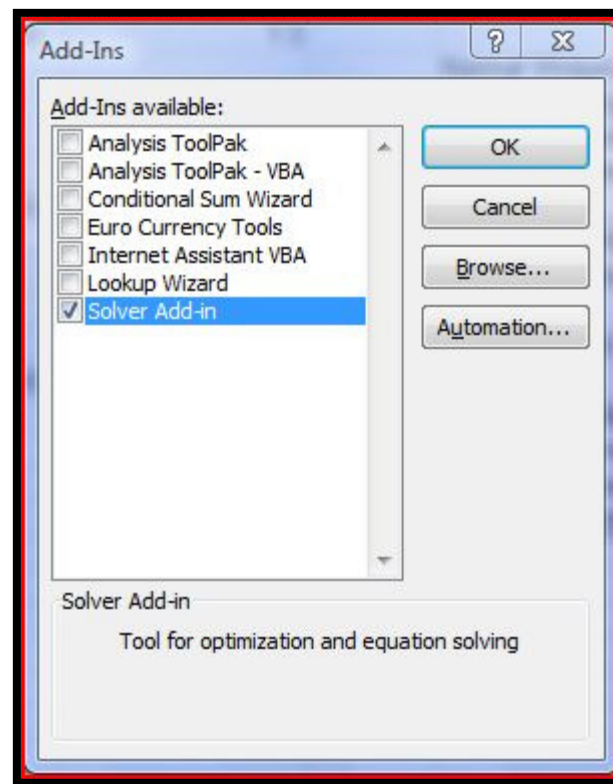
# Activating Solver (Cont.)

- ▶ Click Add-Ins, In the Manage box, choose Excel Add-Ins, select Solver Add-in, Click Go...



# Activating Solver (Cont.)

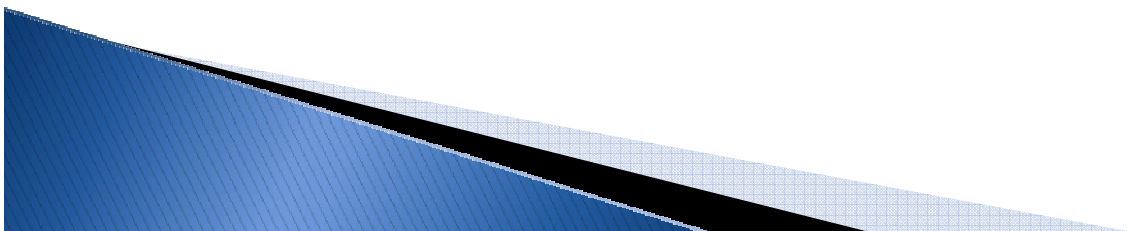
- ▶ Check the Solver Add-in box, and click OK.



# Solver Components

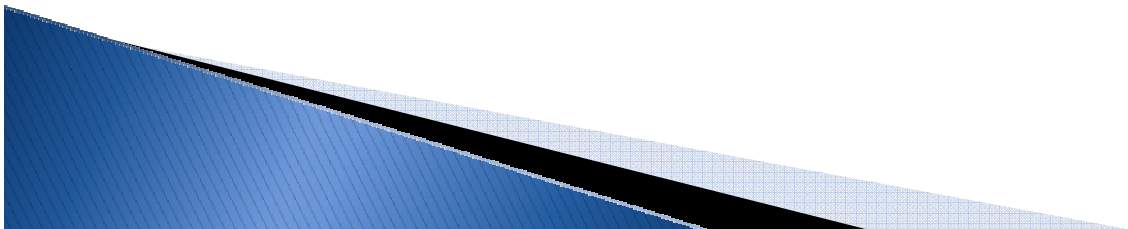
- ▶ Solver has three core components:
  - ✓ Target cell (objective function).
  - ✓ Changing cell (design variables).
  - ✓ Constraints.

The target cell represents the objective or goal that we want to achieve.



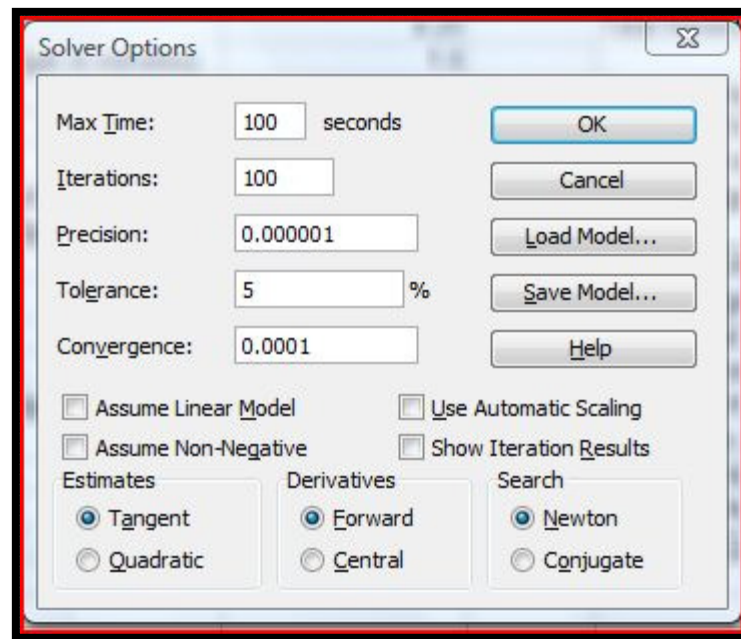
# Solver Components (Cont.)

- ▶ The changing cell represent the range of variables from which the solver selects the specific number(s) that satisfies the optimization.
- ▶ The constraints are restrictions that we impose when constructing the function to be solved.



# What options can we choose?

- ▶ Solver has several options you can play with to optimize your results:

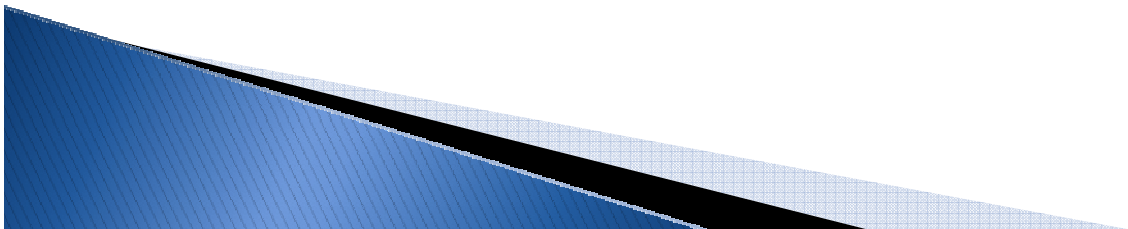


# Finding Roots With Solver

- ▶ Solver requires that the equation be written in a variation of standard form 1.

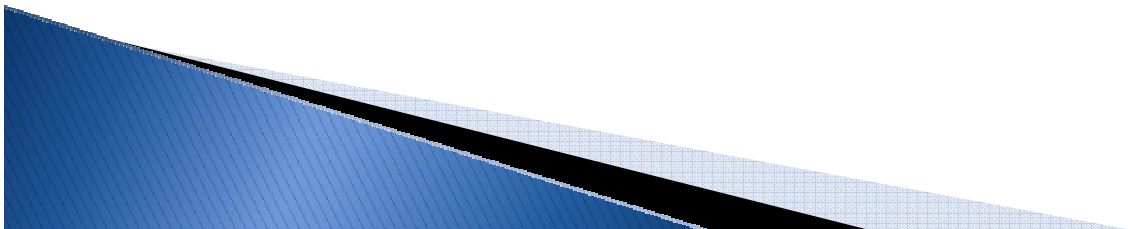
## Procedure:

- Express equation using form 1.
- Set a cell to hold the guess value.
- Set a cell containing the formula (form 1).
- Start the Solver using menu options:  
**Data/Analysis/[Solver]**
- The solver parameter dialog will be displayed.



# Finding Roots with Solver (Cont.)

- ▶ The target cell is the cell containing the formula.
- ▶ Set target cell equal to Zero. By doing this Excel will continue guessing until the formula is equal to zero.
- ▶ Set the value of the field “By Changing Cell” equal to the cell that holds the guess value.



# Finding Roots with Solver (Cont.)

The screenshot shows the Microsoft Excel interface with the Solver Parameters dialog box open. The worksheet contains the following data:

	A	B	C	D	E	F	G	H
55								
56	<b>Using the Solver</b>							
57								
58	Guess:	0.8						
59	f(x):	-1.088						
60								
61								
62								
63								
64								
65								
66								
67								
68								
69								
70								
71								
72								
73								
74								
75								

The Solver Parameters dialog box is configured as follows:

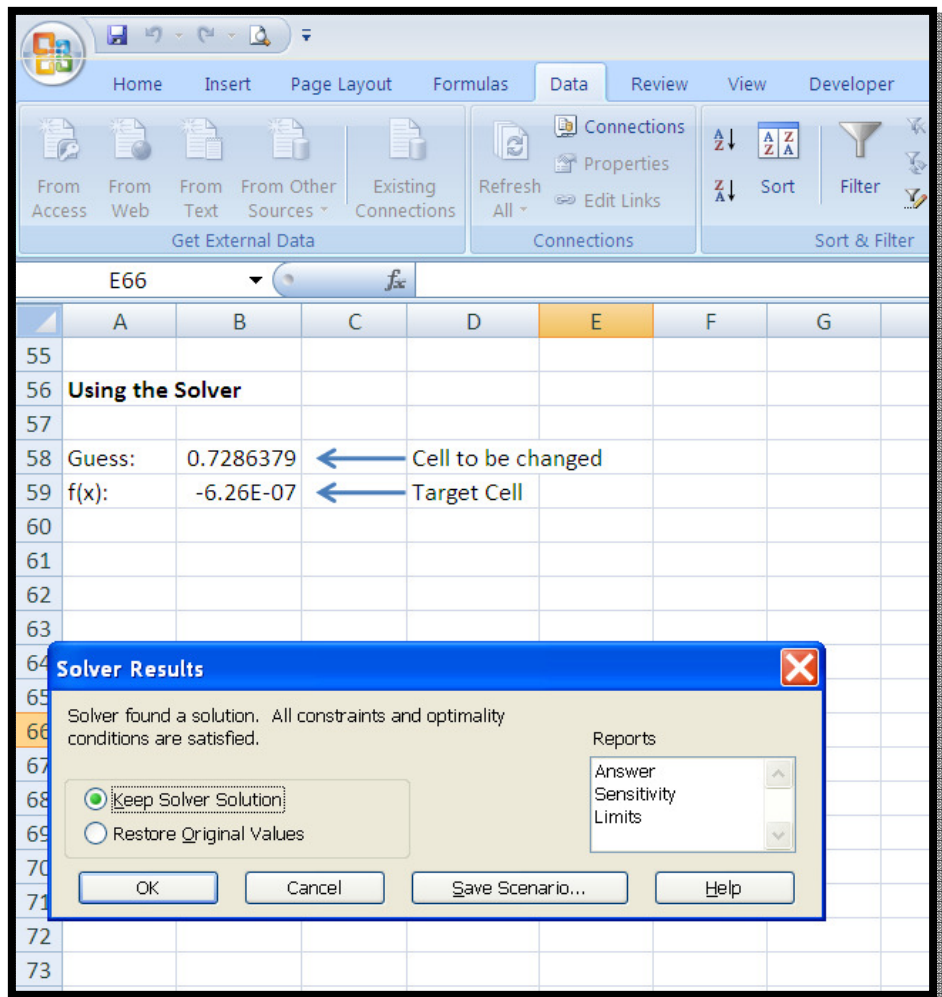
- Set Target Cell:** \$B\$59
- Equal To:**  Value of: 0
- By Changing Cells:** \$B\$58
- Subject to the Constraints:** (Empty list)

Buttons in the dialog box include: Solve, Close, Options, Add, Change, Delete, Reset All, and Help.



# Finding Roots with Solver (Cont.)

- ▶ Once the required information has been set, click on the solve button to iterate for a solution.
- ▶ Here the Solver Results Dialog indicates that a solution was found.
- ▶ To search for other roots, set a new guess value and run the Solver again.

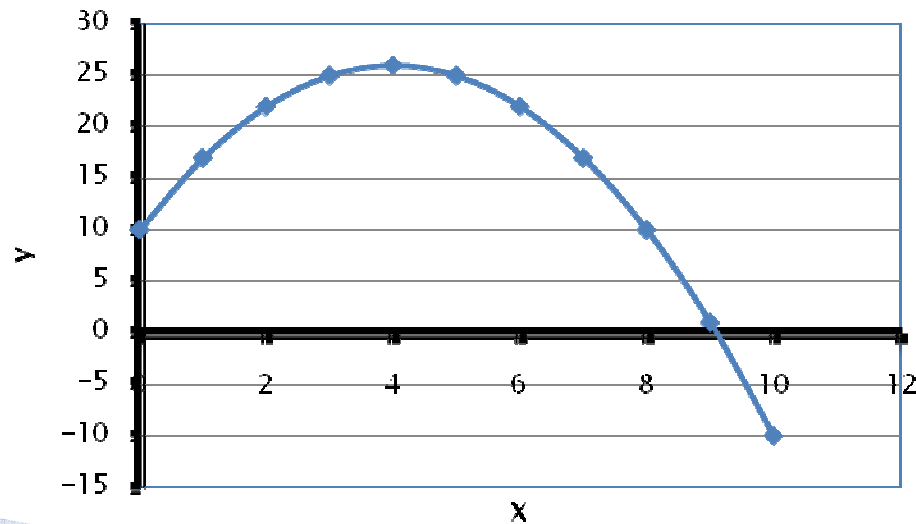


# Optimization Using Solver (Cont.)

- ▶ The Solver can also be used for optimization problems. As a simple example, consider:

$$y = 10 + 8x - x^2$$

This equation has a maximum value of 26 at  $x=4$



# Optimization Using Solver (Cont.)

- ▶ Set the target cell as the cell containing the formula.
- ▶ Set the target cell equal to “Max”.
- ▶ Set changing cell as the cell that holds the guess value.
- ▶ Click solve.

The screenshot shows the Microsoft Excel interface with the Solver Parameters dialog box open. The dialog box is titled "Solver Parameters" and has the following settings:

- Set Target Cell:
- Equal To:  Max  Min  Value of:
- By Changing Cells:

The dialog box also has buttons for "Solve", "Close", "Guess", "Options", "Add", "Change", "Delete", "Reset All", and "Help".

# Optimization Using Solver (Cont.)

- ▶ Here the Solver Results Dialog indicates that a solution was found.

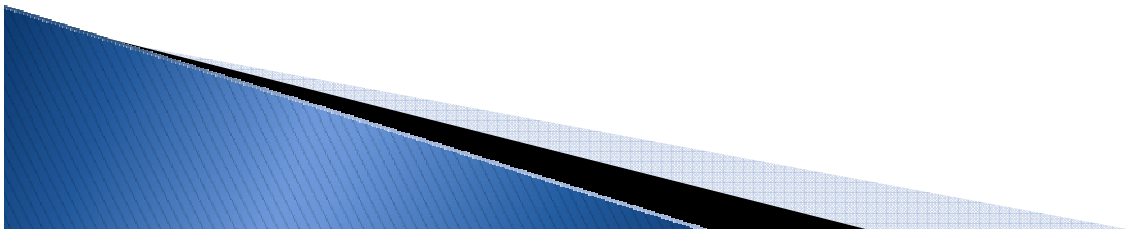
The screenshot displays the Microsoft Excel interface with the Solver Results dialog box open. The dialog box has a blue title bar and contains the following text: "Solver found a solution. All constraints and optimality conditions are satisfied." Below this text are two radio button options: "Keep Solver Solution" (which is selected) and "Restore Original Values". To the right of these options is a "Reports" section with a list box containing "Answer", "Sensitivity", and "Limits". At the bottom of the dialog box are four buttons: "OK", "Cancel", "Save Scenario...", and "Help".

In the background, the Excel spreadsheet is visible. The active cell is B86, which contains the formula  $=10+8*B85-B85^2$ . Cell B85 contains the value 4, and cell B86 contains the result 26. The spreadsheet title bar shows "B86" and the formula bar shows the same formula. The ribbon at the top includes tabs for Home, Insert, Page Layout, Formulas, Data, Review, View, and Developer. The Data tab is active, showing options for "Get External Data" and "Connections".

# Optimization Using Solver (Cont.)

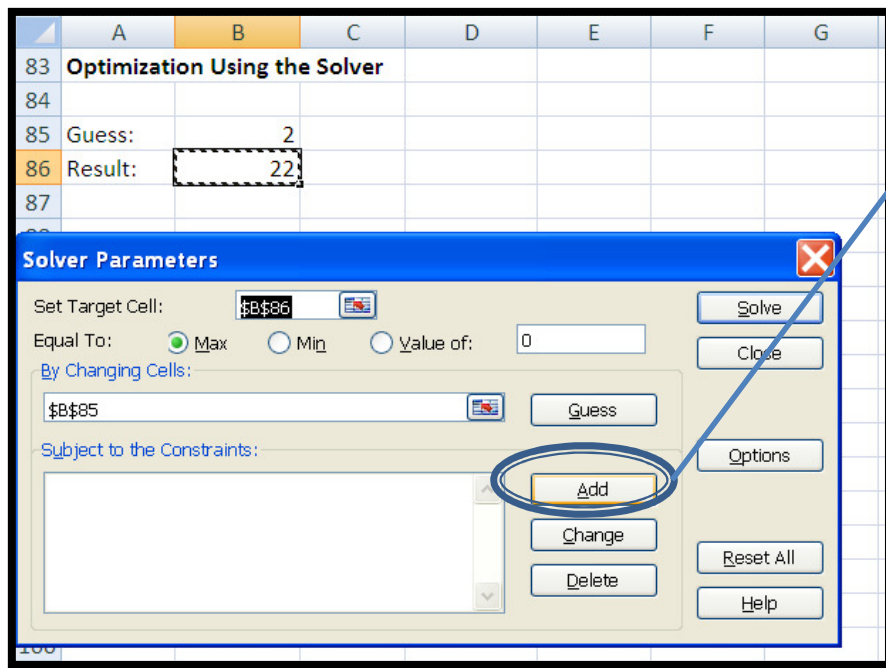
## Adding Constrains:

- ▶ If you want to find the maximum value of  $y$  for  $x \leq 3$ , a constrain must be added.
- ▶ To include a constrain:
  - Click the Add Button at the right of the constrains box.
  - Add constrain.



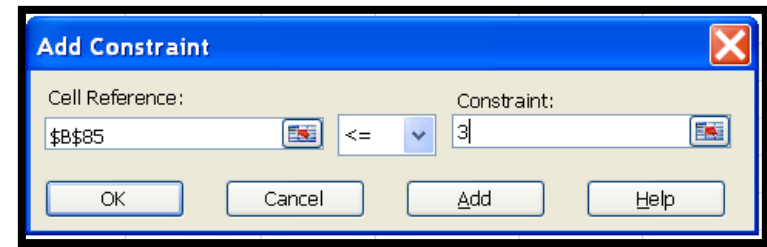
# Optimization Using Solver (Cont.)

## Adding Constrains:

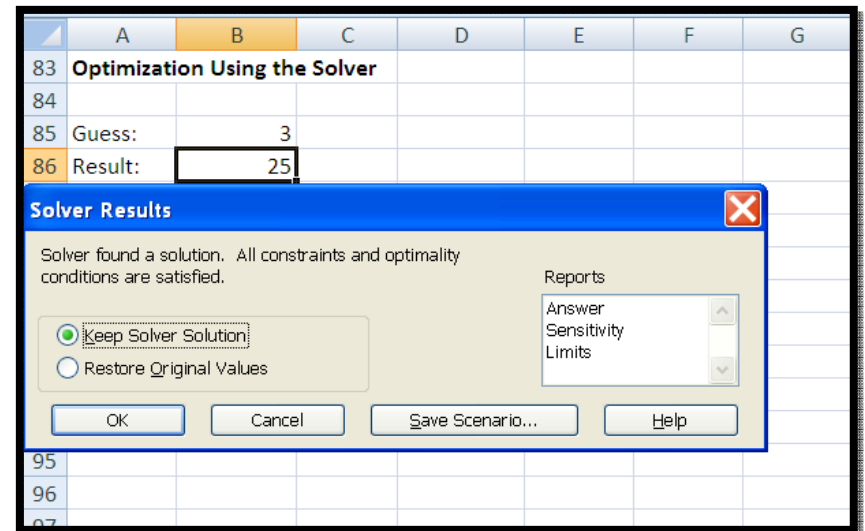


The screenshot shows the Solver Parameters dialog box in Excel. The 'Set Target Cell' is \$B\$86, and the 'By Changing Variable Cells' is \$B\$85. The 'To: Max Of' radio button is selected. The 'Subject to the Constraints' list is empty, and the 'Add' button is circled in blue. An arrow points from the 'Add' button to the 'Add Constraint' dialog box.

	A	B	C	D	E	F	G
83	<b>Optimization Using the Solver</b>						
84							
85	Guess:	2					
86	Result:	22					
87							



The screenshot shows the 'Add Constraint' dialog box. The 'Cell Reference' is \$B\$85, the operator is '<=' (less than or equal to), and the 'Constraint' is 3.



The screenshot shows the Solver Results dialog box. The message states: 'Solver found a solution. All constraints and optimality conditions are satisfied.' The 'Keep Solver Solution' radio button is selected. The 'Reports' list includes 'Answer', 'Sensitivity', and 'Limits'.

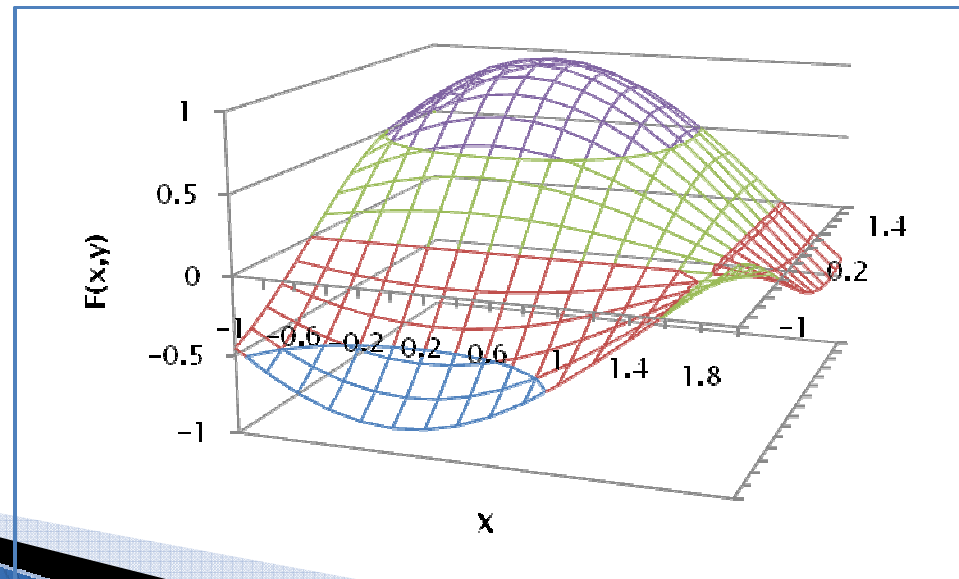
	A	B	C	D	E	F	G
83	<b>Optimization Using the Solver</b>						
84							
85	Guess:	3					
86	Result:	25					

# Optimization Using Solver (Cont.)

## Solving for Multiple Values:

The function:  $f(x, y) = \sin(x) \cdot \cos(y)$

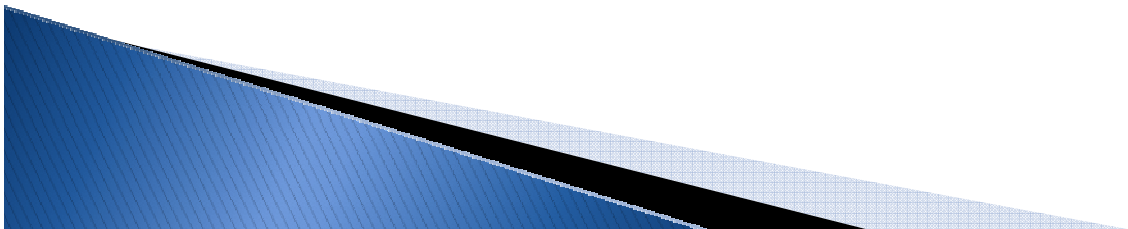
Has a maximum at  $x=1.5708$  and  $y=0$  in the region  $(-1 \leq x \leq 2, -1 \leq y \leq 2)$ .



# Optimization Using Solver (Cont.)

## Solving for Multiple Values:

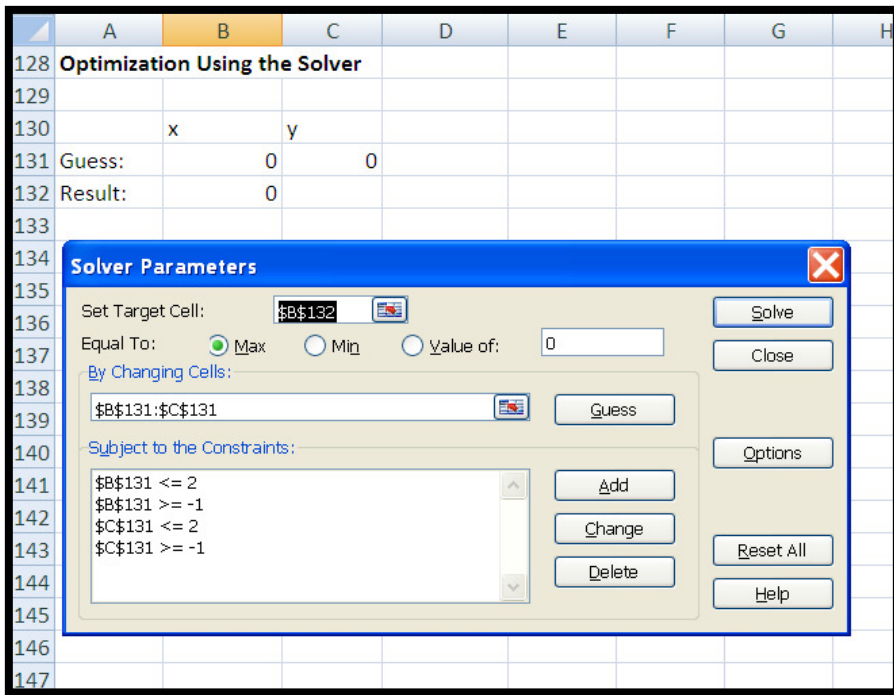
- ▶ The solver will change multiple cells to try to find the requested result.
- ▶ Procedure:
  - Enter guesses for  $x$  and  $y$ , and the equation to be solved.
  - Enter constraints.
  - Click Solve.





# Optimization Using Solver (Cont.)

## Solving for Multiple Values:



The screenshot shows the Solver Parameters dialog box in Excel. The target cell is set to \$B\$132, and the goal is to maximize it. The variable cells are \$B\$131:\$C\$131. The constraints are \$B\$131 <= 2, \$B\$131 >= -1, \$C\$131 <= 2, and \$C\$131 >= -1.

	A	B	C	D	E	F	G	H
128	<b>Optimization Using the Solver</b>							
129								
130		x	y					
131	Guess:		0	0				
132	Result:		0					

**Solver Parameters**

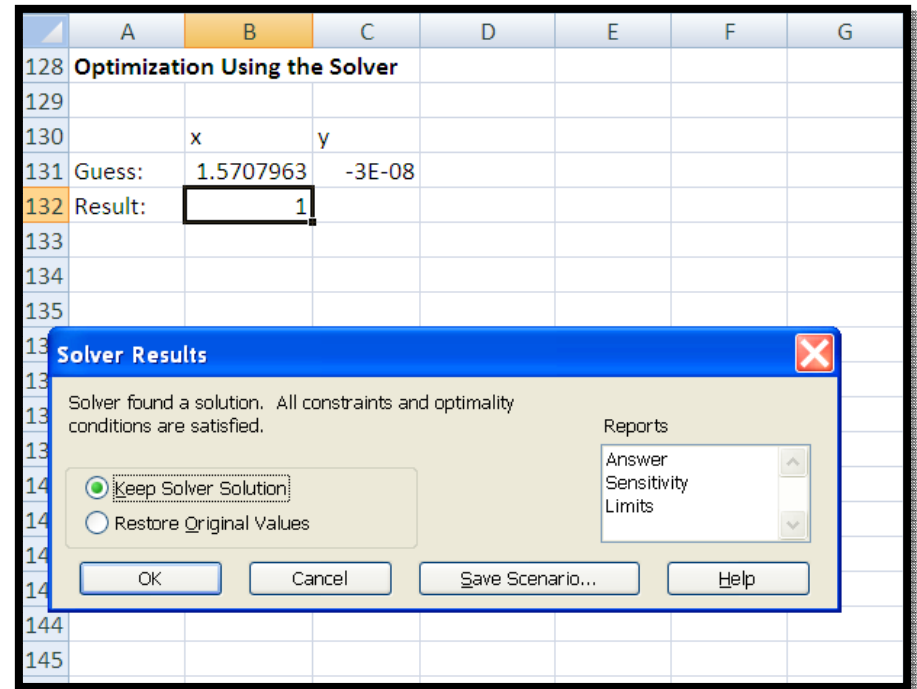
Set Target Cell:

Equal To:  Max  Min  Value of:

By Changing Cells:

Subject to the Constraints:

- 
- 
- 
- 



The screenshot shows the Solver Results dialog box in Excel. The Solver found a solution, and the optimal solution is displayed in the spreadsheet. The variable cells are \$B\$131 and \$C\$131, with values 1.5707963 and -3E-08 respectively. The result cell is \$B\$132, with a value of 1.

	A	B	C	D	E	F	G	H
128	<b>Optimization Using the Solver</b>							
129								
130		x	y					
131	Guess:	1.5707963	-3E-08					
132	Result:	1						

**Solver Results**

Solver found a solution. All constraints and optimality conditions are satisfied.

Reports: Answer, Sensitivity, Limits

Keep Solver Solution  Restore Original Values