EXAMPLE 2 : FINITE DIFFERENCES

Note Title 2/25/2011

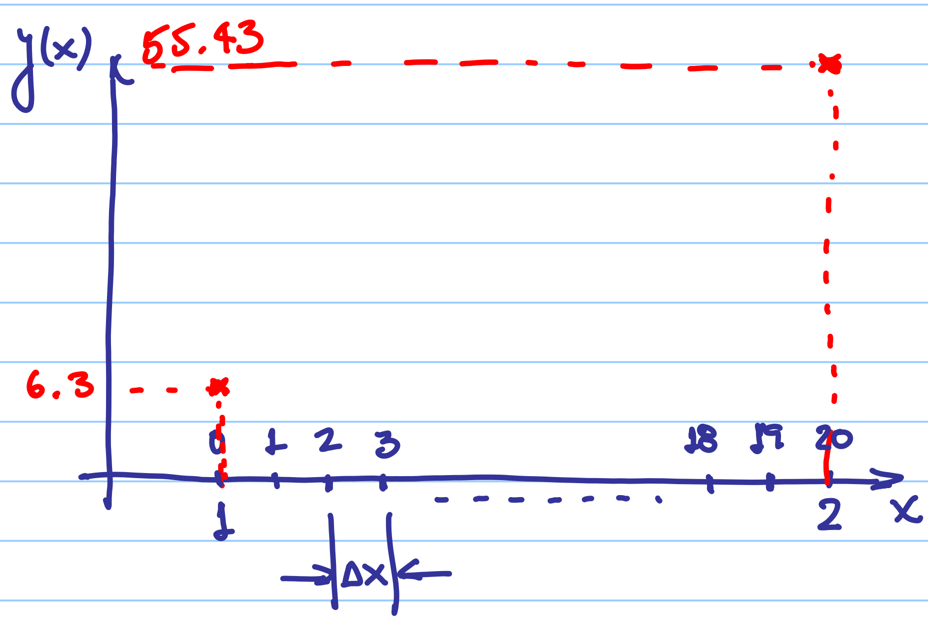
FIND THE SOLUTION OF THE FOLLOWING DIFFERENTIAL EQUATION USING FINITE DIFFERENCES :

$$y'' - y' + y = 3e^{(2x)} - 2\sin x$$

BOUNDARY CONDITIONS : $y(1) = 6.30$

$y(2) = 55.43$

USE $\Delta x = 0.05$



1.- DOMAIN (x) IS DIVIDED IN 20 EQUAL INTERVALS WITH $\Delta x = 0.05$.

2.- FUNCTION $y(x)$ IS KNOWN AT POINT 0 AND POINT 20. IT IS UNKNOWN AT THE REST OF THE POINTS.

3.- IN ORDER TO FIND THE VALUE OF THE FUNCTION $y(x)$ AT THIS POINTS,

APPLY FINITE DIFFERENCE METHOD.
WE KNOW:

$$y''(x_i) \simeq \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta x^2}$$

$$y'(x_i) \simeq \frac{y_{i+1} - y_{i-1}}{2\Delta x}$$

CENTRAL DIFFERENCES

SUBSTITUTE THESE FORMULAS IN THE DIFFERENTIAL EQUATION

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta x^2} - \frac{y_{i+1} - y_{i-1}}{2\Delta x} + y_i = 3e^{(2x_i)} - 2\sin x_i$$

NOTE THAT THIS EQUATION APPLIES AT EVERY POINT "i" OF THE DOMAIN.

IT IS NOT NECESSARY TO APPLY IT FOR POINT "0" AND "20" SINCE THE FUNCTION IS KNOWN AT THESE TWO POINTS. THEN, APPLY THE EQUATION AT THE INTERNAL POINTS (1 TO 19).

RE-WRITE THE EQUATION FIRST IN A SIMPLER FORM:

$$\left(\frac{1}{\Delta x^2} - \frac{1}{2\Delta x}\right) y_{i+1} + \left(1 - \frac{2}{\Delta x^2}\right) y_i + \left(\frac{1}{\Delta x^2} + \frac{1}{2\Delta x}\right) y_{i-1} = 3e^{(2x_i)} - 2\sin x_i$$

THEN

$$\underbrace{C_0\, y_{i-1} + C_1\, y_i + C_2\, y_{i+1}}_{\text{UNKNOWNS}} = \underbrace{3e^{2x_i} - 2\sin x_i}_{\text{KNOWN}}$$

WITH

$$C_0 = \left(\frac{1}{\Delta x^2} + \frac{1}{2\Delta x}\right)$$

$$C_1 = \left(1 - \frac{2}{\Delta x^2}\right)$$

$$C_2 = \left(\frac{1}{\Delta x^2} - \frac{1}{2\Delta x}\right)$$

COEFFICIENTS THAT DEPENDS ON Δx VALUE.

FOR POINT

1 $\quad C_0\, y_0 + C_1\, y_1 + C_2\, y_2 = 3e^{2x_1} - 2\sin x_1$

2 $\quad C_0\, y_1 + C_1\, y_2 + C_2\, y_3 = 3e^{2x_2} - 2\sin x_2$

3 $\quad C_0\, y_2 + C_1\, y_3 + C_2\, y_4 = 3e^{2x_3} - 2\sin x_3$

⋮

19 $\quad C_0\, y_{18} + C_1\, y_{19} + C_2\, y_{20} = 3e^{2x_{19}} - 2\sin x_{19}$