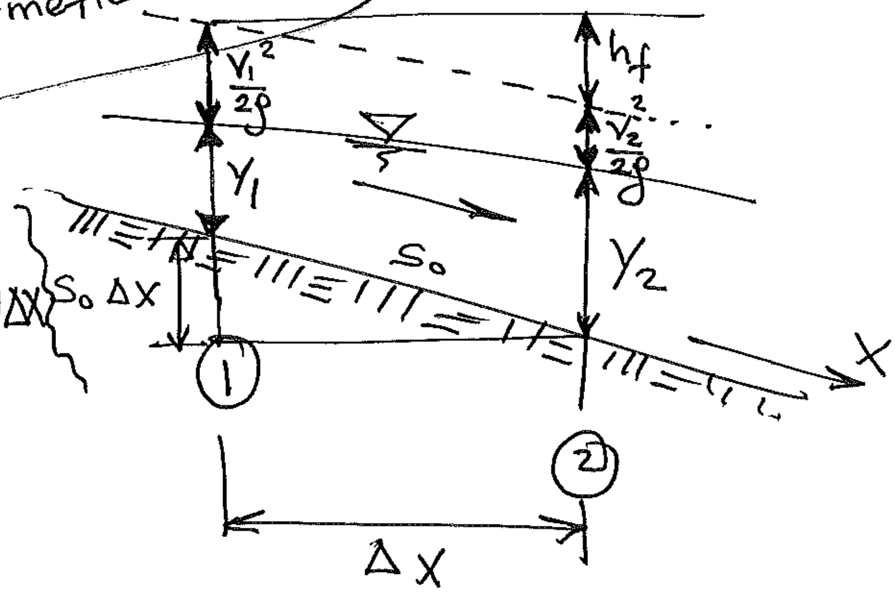
Backwater computation (1) 08/05/03.

Standar step method:

$$E_1 = E_2 + \underbrace{S_f}_{\text{aritmetic mean.}} \Delta X$$

$$E_2 - E_1 = -\frac{(S_{f1}+S_{f2})}{2}\Delta X$$

$$f(y_2) = \underbrace{E_2 - E_1}_{\text{Known}} + \frac{1}{2}(\underbrace{S_{f1}}_{\text{Known}} + S_{f2})\Delta X$$



where:

$$E = y + \frac{V^2}{2g} + Z$$

$$\boxed{\Delta X = X_2 - X_1}$$

Let y_2^* be an estimate of y_2. Based on the Newton Raphson method, $\boxed{\Delta X = \text{Positive}}$ a better estimate of y_2 is obtained from:

$$\boxed{y_2 = y_2^* - \frac{f(y_2^*)}{f'(y_2^*)}}$$

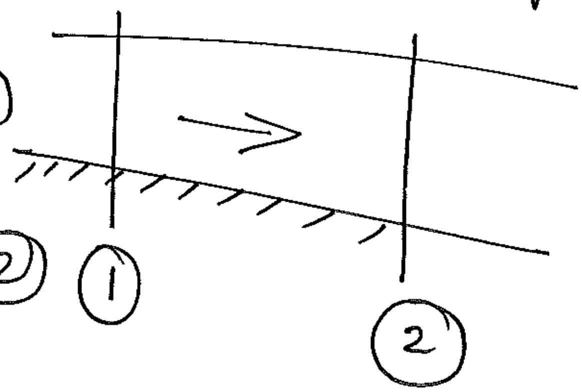
$$\boxed{\frac{df}{dy} = 1 + \frac{d}{dy}\left(\frac{V_2^2}{2g}\right) + \frac{\Delta X}{2}\frac{d S_{f2}}{dy}}$$

Geometric mean friction

② 08/05/03

$$E_1 = E_2 + \bar{S}_f \Delta X$$

$$E_2 - E_1 = -\bar{S}_f \Delta X \quad \cdots ①$$



$$\boxed{\bar{S}_f = \sqrt{S_{f1} \cdot S_{f2}}} \quad \cdots ②$$

② into ①

$$E_2 - E_1 = -\sqrt{S_{f1} \cdot S_{f2}} \cdot \Delta X$$

$$\boxed{f(Y_2) = E_2 - \underbrace{E_1}_{\text{Known}} + \sqrt{\underbrace{S_{f1}}_{\text{Known}} S_{f2}} \, \Delta X}$$

where:

$$E_2 = y_2 + \underbrace{Z_2}_{\text{contant}} + \frac{V_2^2}{2g}$$

$$\frac{df}{dy} = 1 + \frac{d}{dy}\left(\frac{V_2^2}{2g}\right) + \frac{d}{dy}\left(S_{f1}^{1/2} S_{f2}^{1/2} \Delta X\right)$$

$$\boxed{\frac{df}{dy} = 1 + \frac{d}{dy}\left(\frac{V_2^2}{2g}\right) + S_{f1}^{1/2} \Delta X \underbrace{\frac{dS_{f2}^{1/2}}{dy}}}$$

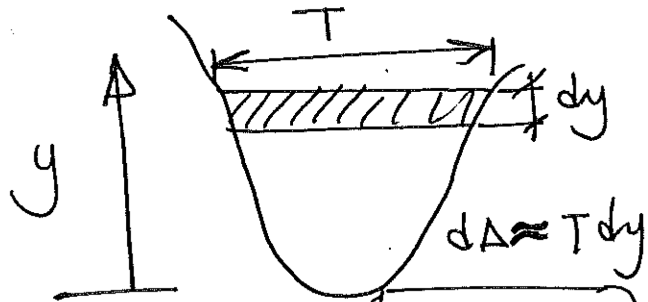
$$\frac{dS_{f2}^{1/2}}{dy} = \frac{1}{2} S_{f2}^{-1/2} \frac{dS_{f2}}{dy} \qquad \frac{\Delta X}{2} S_{f1}^{1/2} S_{f2}^{-1/2} \frac{dS_{f2}}{dy}$$

$$S_{f1}^{1/2} \Delta X \frac{dS_{f2}^{1/2}}{dy} = S_{f1}^{1/2} \Delta X \cdot \frac{1}{2} S_{f2}^{-1/2} \frac{dS_{f2}}{dy} = \frac{\Delta X}{2} S_{f1}^{1/2} \cdot S_{f2}^{-1/2} \frac{dS_{f2}}{dy}$$

③ 08/05/03

$$\frac{d}{dy}\left(\frac{V_2^2}{2g}\right) = \frac{d}{dy}\left(\frac{Q^2}{2g A_2^2}\right) = \frac{Q^2}{2g}\frac{d}{dy}\left(A_2^{-2}\right) = \frac{Q^2}{\cancel{2}g}(-\cancel{2})A_2^{-3}\frac{dA_2}{dy}$$

$$= -\frac{Q^2 T_2}{g A_2^3}$$



$dA \approx T\,dy$

$$\boxed{\frac{dA}{dy} = T}$$

$$\frac{d(S_{f2})}{dy} =$$

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$S = \left[\frac{Qn}{AR^{2/3}}\right]^2 = \left[\frac{Q n P^{2/3}}{A^{5/3}}\right]^2$$

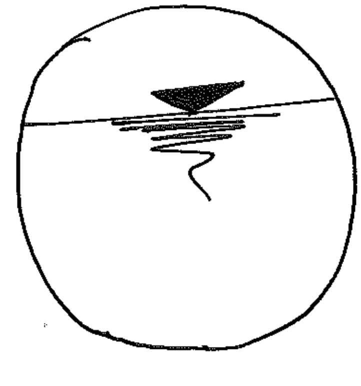
$$\frac{dS_{f2}}{dy} = Q^2 n^2 \frac{d}{dy}\left[P^{4/3}\cdot A^{-10/3}\right] = Q^2 n^2\left[P^{4/3}\frac{dA^{-10/3}}{dy} + A^{-10/3}\frac{dP^{4/3}}{dy}\right]$$

$$= Q^2 n^2\left[P^{4/3}\left(-\frac{10}{3}\right)A^{-\frac{13}{3}}\frac{dA}{dy} + A^{-10/3}\left(\frac{4}{3}\right)P^{1/3}\frac{dP}{dy}\right]$$

T

$2\sqrt{1+z^2}$

for circular cross section



$$P = \frac{1}{2}\theta d_0$$

$$\frac{dP}{dy} = \frac{d_0}{2}\frac{d\theta}{dy}$$

$$\theta = 2\,a\cos\left(1 - \frac{2y}{d_0}\right)$$

$$\frac{d}{dx}\left(a\cos u\right) = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

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$$\frac{d\theta}{dy} = 2\,\frac{(-1)}{\sqrt{1-\left(1-\frac{2y}{d_o}\right)^2}}\left(-\frac{2}{d_o}\right)$$

$$\frac{d\theta}{dy} = \frac{4}{d_o\sqrt{1-\left[1-\frac{2y}{d_o}\right]^2}}$$

$$\boxed{\frac{dP}{dy} = \frac{2}{\sqrt{1-\left[1-\frac{2y}{d_o}\right]^2}}}$$ ✓

$$\therefore \frac{dS_{f2}}{dy} = Q^2 n^2 \underbrace{\left[-\frac{10}{3} P^{4/3} A^{-13/3} T + \frac{8}{3} A^{-10/3} P^{1/3} \left[1-\left(1-\frac{2y}{d_o}\right)^2\right]^{-1/2}\right]}$$

$$\Rightarrow \frac{df}{dy} = 1 - \frac{Q^2 T_2}{g A_2^3} + \frac{\Delta x}{2}\left(Q^2 n^2\right)\left[\qquad\right]$$

 08/05/03

Normal depth

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$y_n = y_n^{*} - \frac{f(y^{*})}{f'(y^{*})} \qquad \boxed{f = Q - \frac{K_s}{n} A R^{2/3} S^{1/2}}$$

$$\frac{df}{dy} = -\frac{K_s}{n} S^{1/2} \frac{d}{dy}\left(A R^{2/3}\right) = \frac{K_s S^{1/2}}{n} \frac{d}{dy}\left[A^{5/3} P^{-2/3}\right]$$

$$\frac{df}{dy} = -\frac{S^{1/2}(K_s)}{n}\left[A^{5/3}\left(-\frac{2}{3}\right)P^{-5/3}\frac{dP}{dy} + P^{-2/3}\left(\frac{5}{3}\right)A^{2/3}\frac{dA}{dy}\right]$$

$$\frac{df}{dy} = -\frac{S^{1/2}K_s}{n}\left[-\frac{2}{3}A^{5/3}P^{-5/3}\frac{dP}{dy} + \frac{5}{3}A^{2/3}P^{-2/3}T\right] \quad \ldots (*)$$

for circular cross section

$$\frac{df}{dy} = -\frac{S^{1/2}K_s}{n}\left[-\frac{2}{3}A^{5/3}P^{-5/3}(2)\left[1-\left(1-\frac{2y}{d_o}\right)^2\right]^{-1/2} + \frac{5}{3}A^{2/3}P^{-2/3}T\right]$$

$$\frac{df}{dy} = -\frac{S^{1/2}K_s}{n}\left[-\frac{4}{3}A^{5/3}P^{-5/3}\left\{1-\left[1-\frac{2y}{d_o}\right]^2\right\}^{-1/2} + \frac{5}{3}A^{2/3}P^{-2/3}T\right]$$

$K_s = 1$ (metric units)

$K_s = 1.49$ (English units).

for trapezoidal cross section

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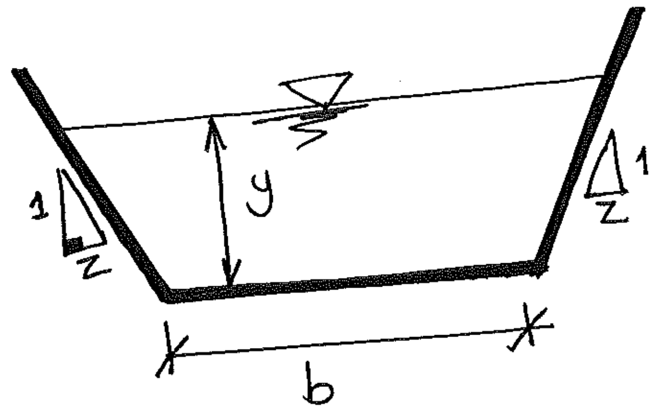
$$P = b + 2y\sqrt{1+z^2}$$



$$\boxed{\frac{dP}{dy} = 2\sqrt{1+z^2}}$$

In (*)

$$\frac{df}{dy} = -\frac{S^{1/2}}{n}\left[-\frac{2}{3}R^{5/3}\left(2\sqrt{1+z^2}\right) + \frac{5}{3}R^{2/3}T\right]$$

$$\boxed{\frac{df}{dy} = -\frac{S^{1/2}}{n}\left[-\frac{4}{3}R^{5/3}\sqrt{1+z^2} + \frac{5}{3}R^{2/3}T\right]}$$

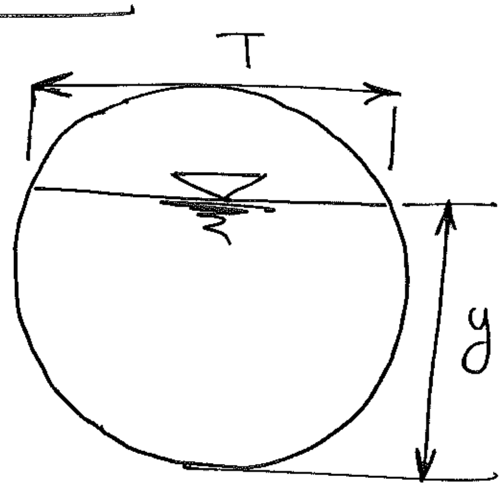
CRITICAL DEPTH

$$\frac{Q^2 T}{g A^3} = 1 \qquad f = 1 - \frac{Q^2 T}{g A^3}$$

$$\frac{df}{dy} = -\frac{Q^2}{g}\frac{d}{dy}\left[A^{-3}T\right] = -\frac{Q^2}{g}\left[A^{-3}\frac{dT}{dy} + T(-3)A^{-4}\frac{dA}{dy}\right]$$

$$\boxed{= -\frac{Q^2}{g}\left[A^{-3}\frac{dT}{dy} - 3TA^{-4}\underbrace{\frac{dA}{dy}}_{T}\right]}$$

<u>for circular cross section</u>



$$T = 2\sqrt{y(d_o - y)}$$

$$\frac{dT}{dy} = \cancel{2}\times\frac{1}{\cancel{2}}\left[y(d_o-y)\right]^{-1/2}(d_o-2y)$$

$$\boxed{\frac{dT}{dy} = (d_o-2y)\left[y(d_o-y)\right]^{-1/2}}$$

$$\therefore \boxed{\frac{df}{dy} = -\frac{Q^2}{g}\left\{A^{-3}(d_o-2y)\left[y(d_o-y)\right]^{-1/2} - 3T^2A^{-4}\right\}}$$

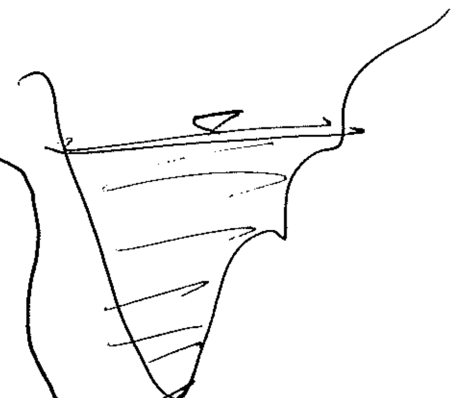
for trapezoidal cross section

$$T = b + 2zy$$

$$\frac{dT}{dy} = 2Z$$

$$\boxed{\therefore \frac{df}{dy} = -\frac{Q^2}{g}\left[2ZA^{-3} - 3T^2A^{-4}\right]}$$

$$y_c = y_c^* - \frac{f(y_c^*)}{f'(y_c^*)}$$



$$\boxed{y_c = y_c^* - \frac{\left[1 - \frac{Q^2 T}{gA^3}\right]^*}{-\frac{Q^2}{g}\left[2ZA^{-3} - 3T^2A^{-4}\right]}}$$