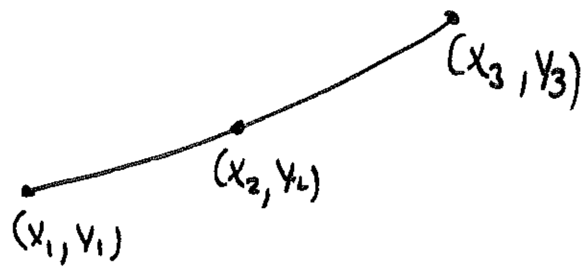
Parabolic interpolation



$$(y - y_3) = a(x - x_3)^2 + b(x - x_3) + c$$

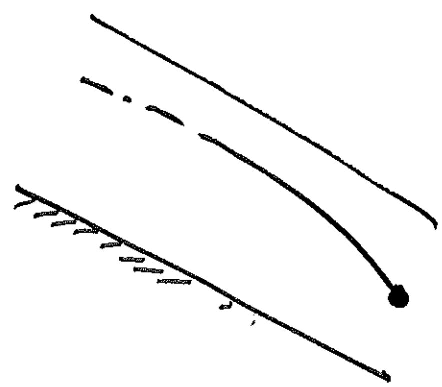
with $P_1(x_1, y_1)$

$$(y_1 - y_3) = a(x_1 - x_3)^2 + b(x_1 - x_3) + c \quad \ldots \quad \alpha$$

with $P_2(x_2, y_2)$

$$(y_2 - y_3) = a(x_2 - x_3)^2 + b(x_2 - x_3) + c \quad \ldots \quad \beta$$

with $P_3(x_3, y_3)$



$$0 = a(0) + b(0) + c \longrightarrow c = 0$$

from α $\quad b = \dfrac{(y_1 - y_3) - a(x_1 - x_3)^2}{x_1 - x_3}$

In β

$$y_2 - y_3 = a(x_2 - x_3)^2 + \underbrace{\frac{x_2 - x_3}{x_1 - x_3}}_{\theta}\left[y_1 - y_3 - a(x_1 - x_3)^2\right]$$

$$\boxed{\theta = \frac{x_2 - x_3}{x_1 - x_3}}$$

$$y_2 - y_3 = a(x_2 - x_3)^2 + \theta(y_1 - y_3) - a\theta(x_1 - x_3)^2$$

$$\underbrace{(y_2 - y_3) - \theta(y_1 - y_3)}_{temp_1} = a\left[(x_2 - x_3)^2 - \theta(x_1 - x_3)^2\right]$$

$temp_1 = y_2 - y_3 - \theta(y_1 - y_3)$

$temp_2 = (x_2 - x_3)^2 - \theta(x_1 - x_3)^2$

$$\therefore\ a = \frac{temp\,1}{temp\,2}$$

from ⓓ
$$b = \frac{(y_1 - y_3) - a(x_1 - x_3)^2}{x_1 - x_3}$$

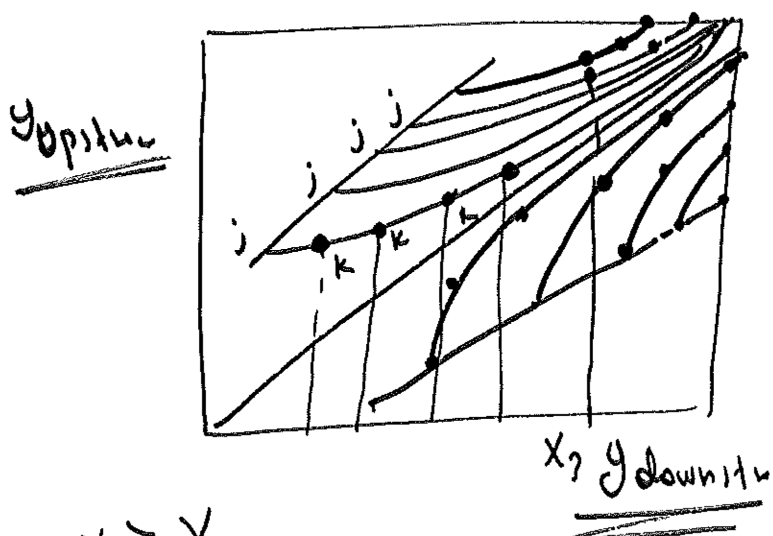
$$temp\ 3 = (y_1 - y_3) - a(x_1 - x_3)^2$$
$$temp\ 4 = x_1 - x_3$$

$$\therefore \boxed{b = \frac{temp\ 3}{temp\ 4}}$$

$$\therefore \boxed{(y - y_3) = a(x - x_3)^2 + b(x - x_3)}$$

$m = y_4 - y_3 \quad y_4 > y_3$

$$m = a\underbrace{(x - x_3)}_{U}^2 + b\underbrace{(x - x_3)}_{U}$$



$$m = aU^2 + bU$$
$$aU^2 + bU - m = 0$$
$$U = \frac{-b \pm \sqrt{b^2 - 4a(-m)}}{2a}$$
$$U = \frac{-b \pm \sqrt{b^2 + 4am}}{2a}$$

$x > x_3$

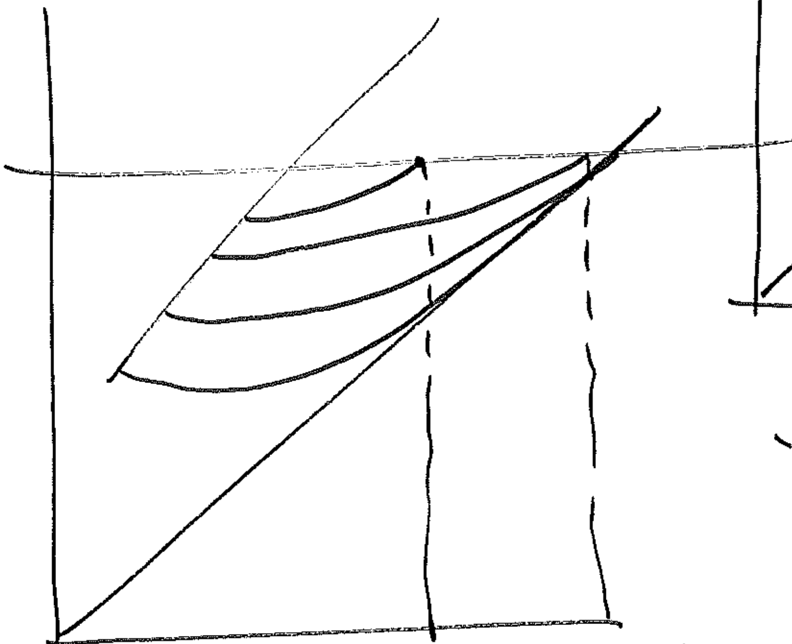
U should be positive

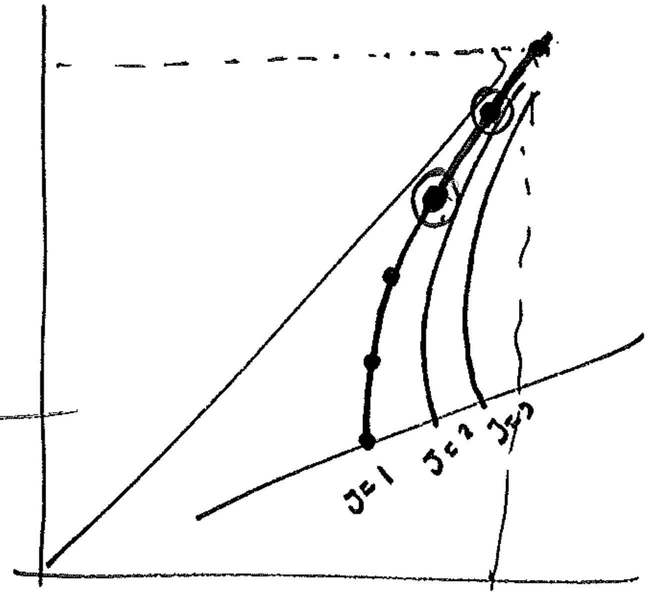
Then U negative does not have sense

$$\therefore U = \frac{-b + \sqrt{b^2 + 4am}}{2a}$$

$$\therefore x - x_3 = U$$
$$\therefore \boxed{x = U + x_3}$$

$q(J) = 40$ ③

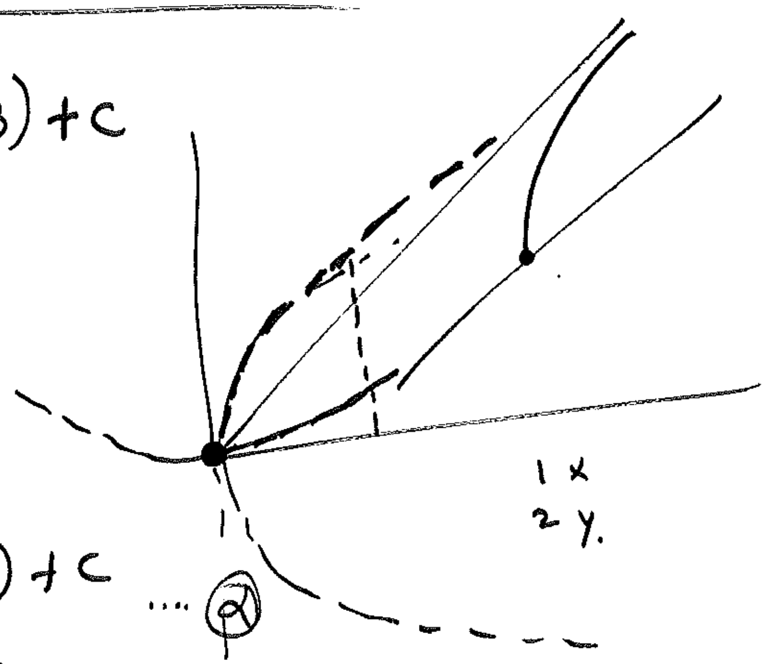




①, ②, ③

For steep slope

$$x - x_3 = a(y - y_3)^2 + b(y - y_3) + c$$



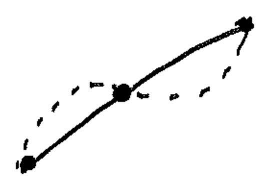
with $P_2(x_3, y_3)$

$$0 = 0 + 0 + c \longrightarrow c = 0$$

with $P_1(x_1, y_1)$

$$x_1 - x_3 = a(y_1 - y_3)^2 + b(y_1 - y_3) + c \quad \ldots \alpha$$

$$x_2 - x_3 = a(y_2 - y_3)^2 + b(y_2 - y_3) \quad \ldots \beta$$



from α

$$\boxed{b = \frac{x_1 - x_3 - a(y_1 - y_3)^2}{y_1 - y_3}}$$

In β

$$x_2 - x_3 = a(y_2 - y_3)^2 + \left[\frac{x_1 - x_3 - a(y_1 - y_3)^2}{y_1 - y_3}\right] y_2 - y_3$$

$$x_2 - x_3 = a(y_2 - y_3)^2 + \theta(x_1 - x_3) - a\theta(y_1 - y_3)^2 \qquad \boxed{\theta = \frac{y_2 - y_3}{y_1 - y_3}}$$

$$\underbrace{x_2 - x_3 - \theta(x_1 - x_3)}_{\text{temp 1}} = a\Big[\underbrace{(y_2 - y_3)^2 - \theta(y_1 - y_3)^2}_{\text{temp 2}}\Big]$$

$\therefore$

$$a = \frac{\text{temp 1}}{\text{temp 2}}$$

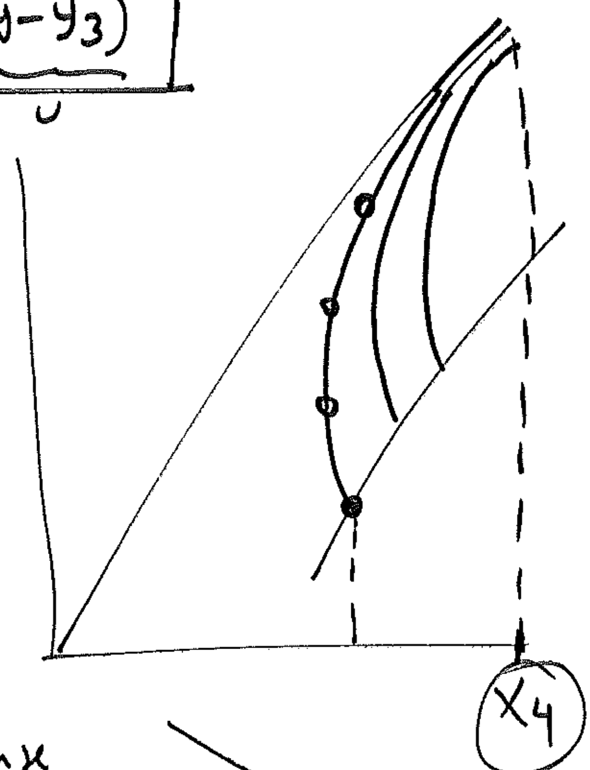
From ⓐ $b = \dfrac{\overbrace{x_1 - x_3 - a(y_1-y_3)^2}^{\text{temp 3}}}{\underbrace{y_1 - y_3}_{\text{temp 4}}}$

$temp\ 3 = x_1 - x_3 - a(y_1 - y_3)^2$

$temp\ 4 = y_1 - y_3$

$\therefore b = \dfrac{temp\ 3}{temp\ 4}.$

$$\therefore \boxed{x - x_3 = a\underbrace{(y-y_3)}_{u}{}^2 + b\underbrace{(y-y_3)}_{u}}$$



$m = x_4 - x_3$

$m = au^2 + bu$

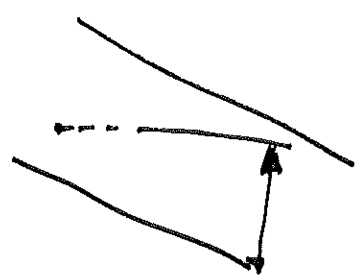
$au^2 + bu - m$

$u\ (+)$

$m\ (+)$

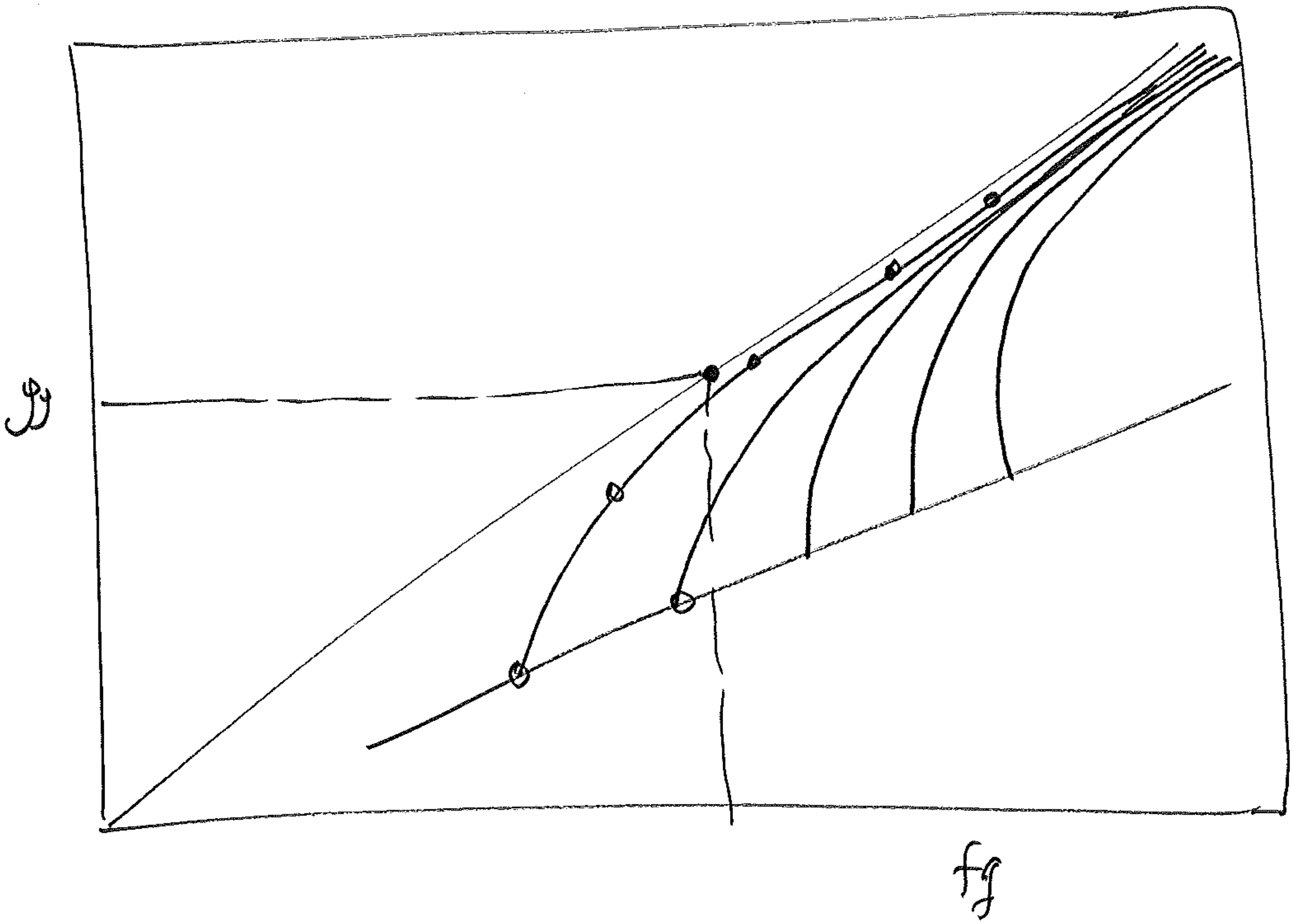
$\therefore u = \dfrac{-b \pm \sqrt{b^2+4am}}{2a}$

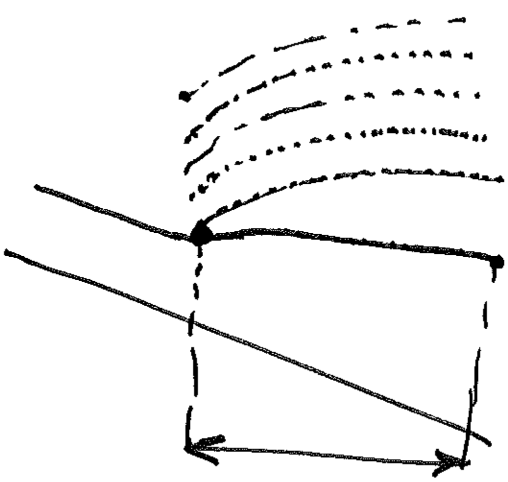
it does not have sense

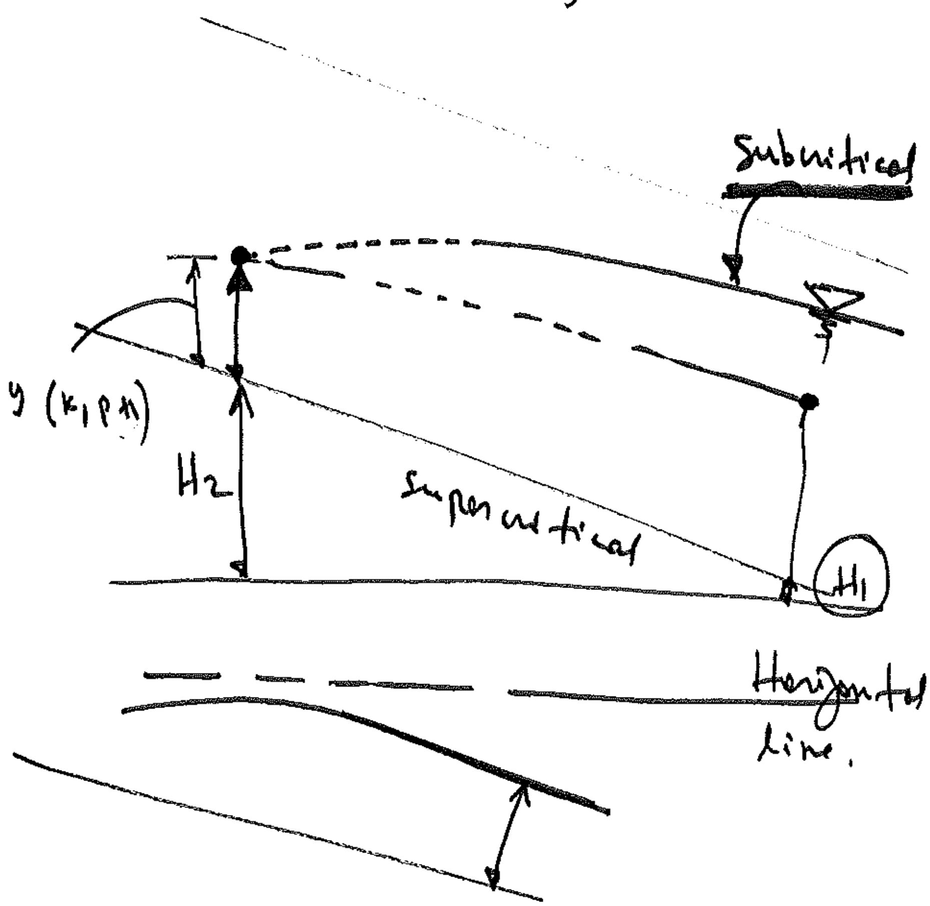


$$\therefore \boxed{u = \dfrac{-b + \sqrt{b^2+4am}}{2a}}$$

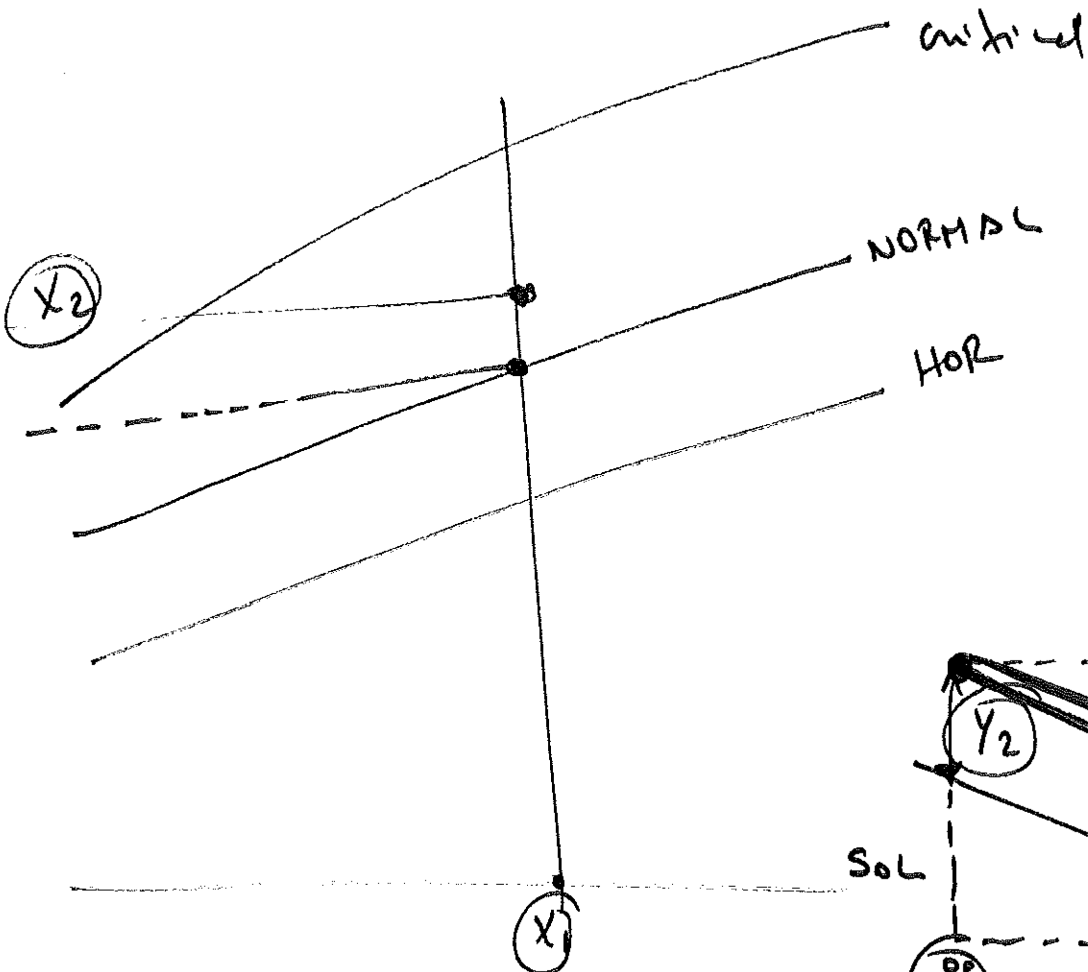
$y_4 - y_3 = \dfrac{-b + \sqrt{b^2+4am}}{2a} \rightarrow y_4 = \dfrac{-b + \sqrt{b^2+4am}}{2a} + y_3$

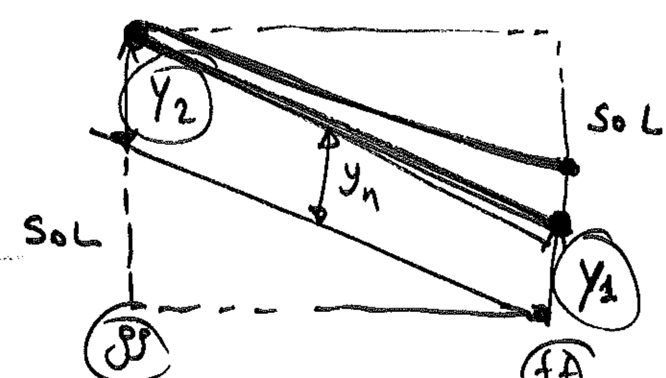






check horizontal and normal line





if $gg > y_n \rightarrow M_1$

If $gg < y_n \rightarrow M2$

what I should do to locate the three zones