08/06/03

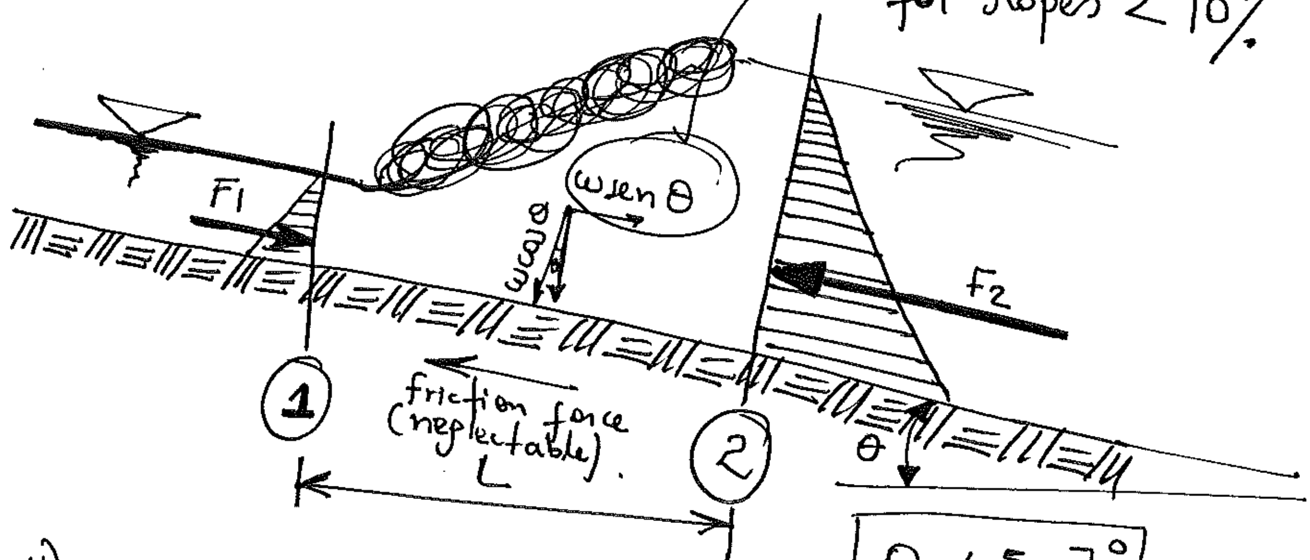
Hydraulic Jump

for Trapezoidal channels

It is not applicable for channels > 10%

$\boxed{\text{Slopes} < 10\%}$

neglectable for slopes < 10%



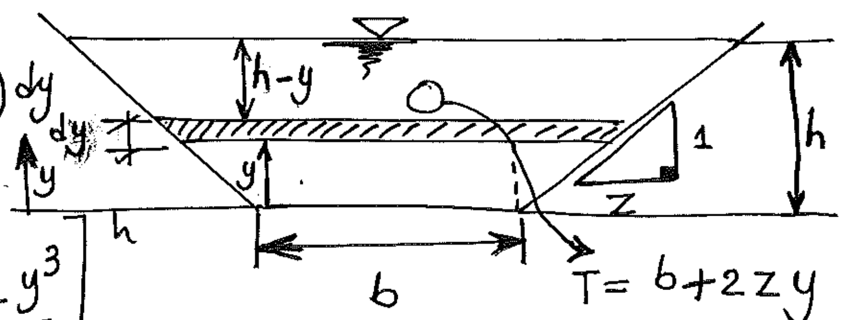
$\boxed{\theta < 5.7°}$

$$P = \gamma(h-y)$$

$$F = \int_0^h \gamma(h-y)\,dA$$

$$dA = (b+2zy)\,dy$$

$$\therefore F = \int_0^h \gamma(h-y)(b+2zy)\,dy$$



$$F = \int_0^h \gamma h(b+2zy)\,dy - \int_0^h \gamma y(b+2zy)\,dy$$

$$F = \gamma h\left[by + \cancel{2}z\frac{y^2}{\cancel{2}}\right]_0^h - \gamma\left[b\frac{y^2}{2} + 2z\frac{y^3}{3}\right]_0^h$$

$$F = \gamma h\left[bh + zh^2\right] - \gamma\left[\frac{bh^2}{2} + \frac{2z}{3}h^3\right] = \frac{\gamma b h^2}{2} + \frac{1}{3}\gamma z h^3$$

②-③ 08/06/23

Momentum (second law of Newton)

$$\rho Q(\beta_2 V_2 - \beta_1 V_1) = F_1 - F_2 + \overbrace{\cancel{W\sin\theta}}^{\approx 0} - \overbrace{\cancel{F_f}}^{\approx 0} \quad \text{friction force} \quad \ldots (1)$$

$$\beta_2, \beta_1 \approx 1.$$

$$\rho Q(V_2 - V_1) = \left(\frac{\gamma b_1}{2} h_1^2 + \frac{1}{3}\gamma z_1 h_1^3\right) - \left(\frac{\gamma b_2}{2} h_2^2 + \frac{1}{3}\gamma z_2 h_2^3\right)$$

$$\gamma = \rho g$$

$$QV_2 + g\left(\frac{b_2 h_2^2}{2} + \frac{1}{3} z_2 h_2^3\right) = \underbrace{QV_1 + g\left(\frac{b_1 h_1^2}{2} + \frac{1}{3} z_1 h_1^3\right)}_{\text{Know}}$$

$$V_2 = \frac{Q}{A_2} \qquad A_2 = (b_2 + z_2 y_2) y_2 \qquad h_2 \to y_2, \quad h_1 \to y_1$$

$$\underbrace{\frac{Q^2}{(b_2 + z_2 y_2) y_2} + g\Bigg[\underbrace{\frac{b_2 y_2^2}{2} + \frac{1}{3} z_2 y_2^3}_{\bar{y}_2}\Bigg] - \Bigg(QV_1 + g\Big(\underbrace{\frac{b_1 y_1^2}{2} + \frac{1}{3} z_1 y_1^3}_{\bar{y}_1}\Big)\Bigg)}_{f} \quad \ldots \quad = QV_1 + g\left(\frac{b_1 y_1^2}{2} + \frac{1}{3} z_1 y_1^3\right)$$

$$f = 0$$

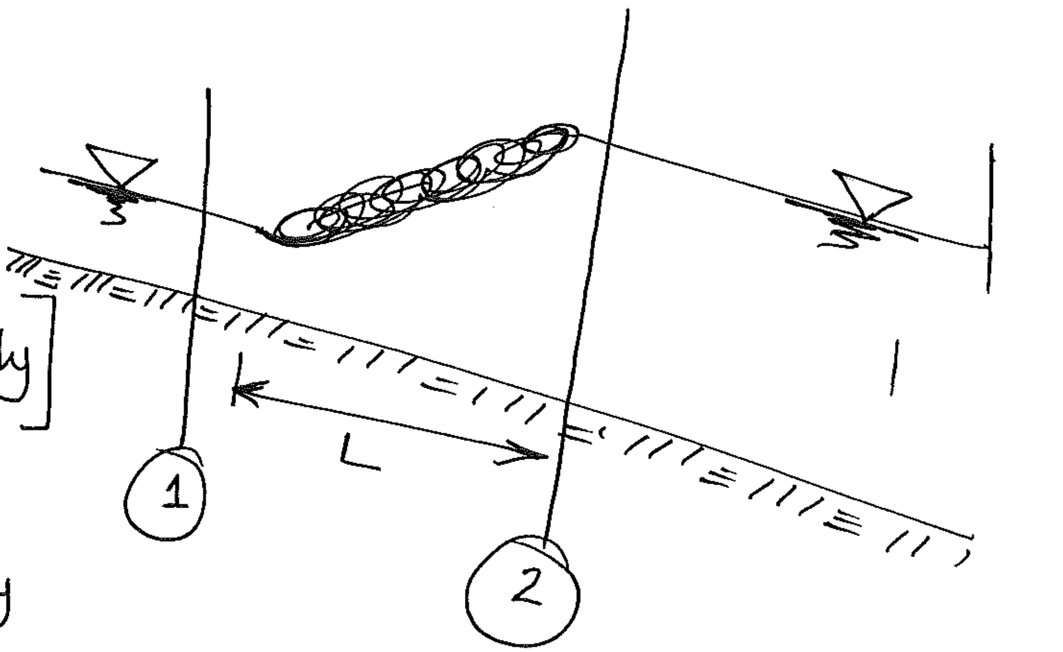
$$\frac{df}{dy_2} = -\frac{Q^2 z_2}{y_2 (b_2 + z_2 y_2)^2} - \frac{Q^2}{y_2^2 (b_2 + z_2 y_2)} + g\left(b_2 y_2 + y_2^2 z_2\right) \quad \text{known}$$

$$\therefore \underbrace{y_2^{**}}_{\text{better approximation}} = \overbrace{y_2^{*}}^{\text{guess}} - \frac{f}{df/dy_2} \quad \Bigg\} \text{ Newton Raphson iteration}$$

Hydraulic jump for Circular Channels

④ 08/06/03

08/05/03

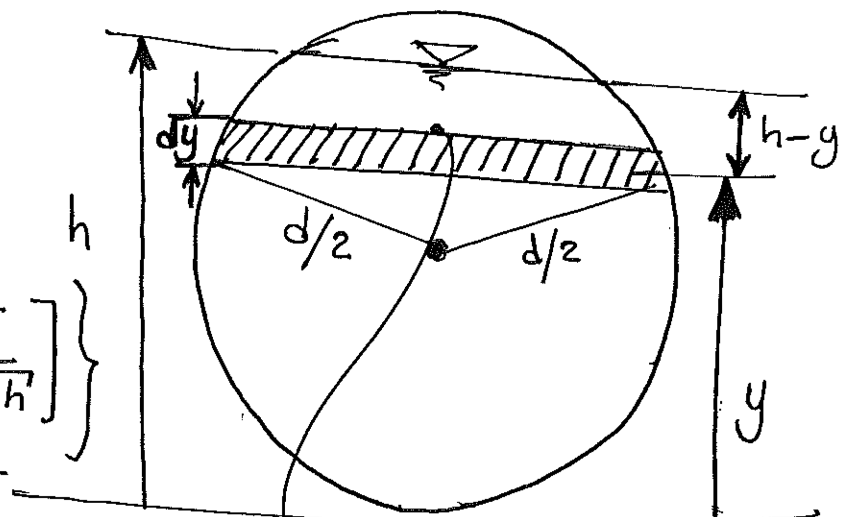


$$P = \gamma (h-y)$$

$$F = \int_0^h P\,dA$$

$$F = \int_0^h \gamma (h-y)\left[2\sqrt{y(d_o-y)}\,dy\right]$$

$$F = \gamma\left[2h\int_0^h \sqrt{y(d_o-y)}\,dy - \int_0^h 2y\sqrt{y(d_o-y)}\,dy\right]$$



Integrating

$$F = \frac{\gamma\left\{(d-h)h(3d^2-4dh+4h^2) - 3d^2(d-2h)\sqrt{d-h}\,\sqrt{h}\,\mathrm{ArcTan}\left[\frac{\sqrt{h}}{\sqrt{d-h}}\right]\right\}}{12\sqrt{(d-h)h}}$$

$$T = 2\sqrt{y(d_o-y)}$$

$$\boxed{dA = 2\sqrt{y(d_o-y)}\,dy}$$

In 1

$$\rho Q(V_2 - \underset{\text{Know}}{V_1}) = \underset{\text{Know}}{F_1} - F_2$$

$$\rho Q V_2 + \frac{\gamma F_2}{\gamma} = \frac{\gamma F_1}{\gamma} + QV_1 \rightarrow QV_2 + g\frac{F_2}{\gamma} = \underbrace{g\frac{F_1}{\gamma} + QV_1}_{\text{Know}}$$

08/06/03

$$\frac{df}{dy} = \frac{d}{dy}\left(Q \cdot \frac{Q}{A_2}\right) + g\,\frac{d}{dy}\left(\frac{F_2}{\gamma}\right) = Q^2(-1)A_2^{-2}\underbrace{\frac{dA_2}{dy}}_{T} + g\,\frac{d}{dy}\left(\frac{F_2}{\gamma}\right)$$

$$= -Q^2 A_2^{-2}\, T + g\,\frac{d}{dy}\left(\frac{F_2}{\gamma}\right)$$

$$\frac{df}{dy} = -Q^2 A_2^{-2}\, T + 9.8\left[\frac{-(d-2h)(d-h)h + d^2\sqrt{d-h}\,\sqrt{h}\,\mathrm{Arctan}\left[\frac{\sqrt{h}}{\sqrt{d-h}}\right]}{2\sqrt{(d-h)h}}\right]$$

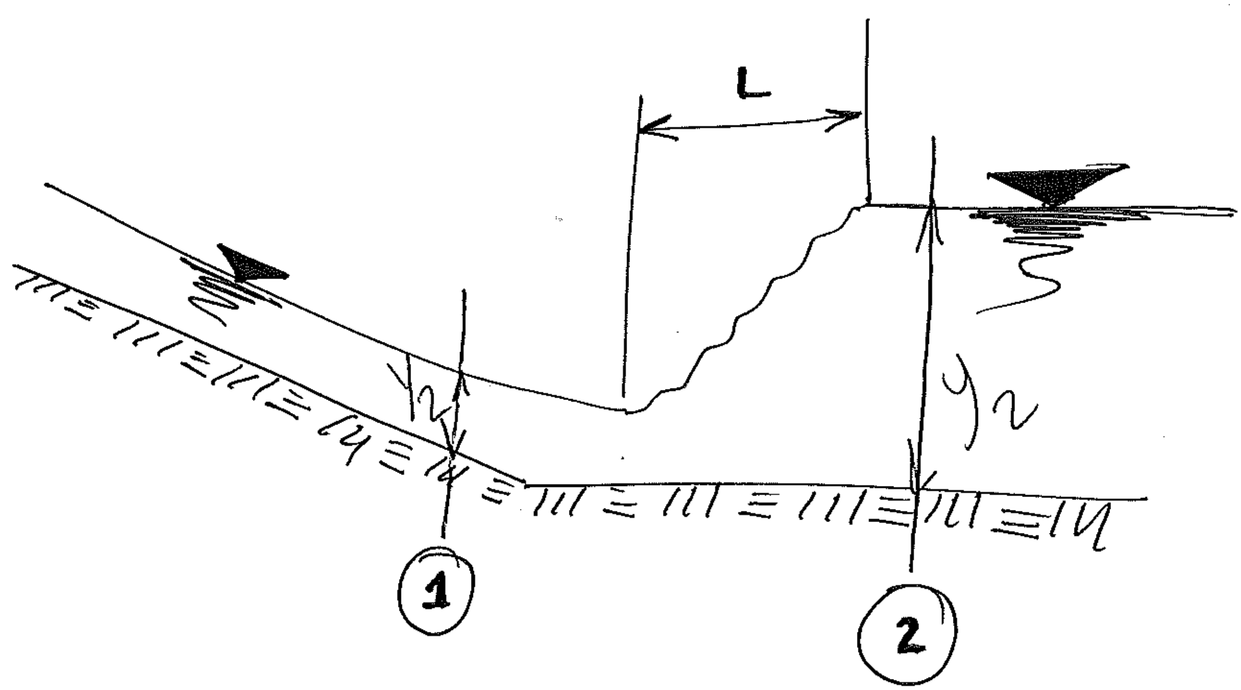
<u>Energy law</u>

$$\Delta E = E_1 - E_2 = \underbrace{y_1 + \frac{V_1^2}{2g} + Z_1}_{\text{Known}} - \underbrace{\left[y_2 + \frac{V_2^2}{2g} + Z_2\right]}_{\text{computed}}$$

Length of the hydraulic jump (Applicable to all of the cross sections).

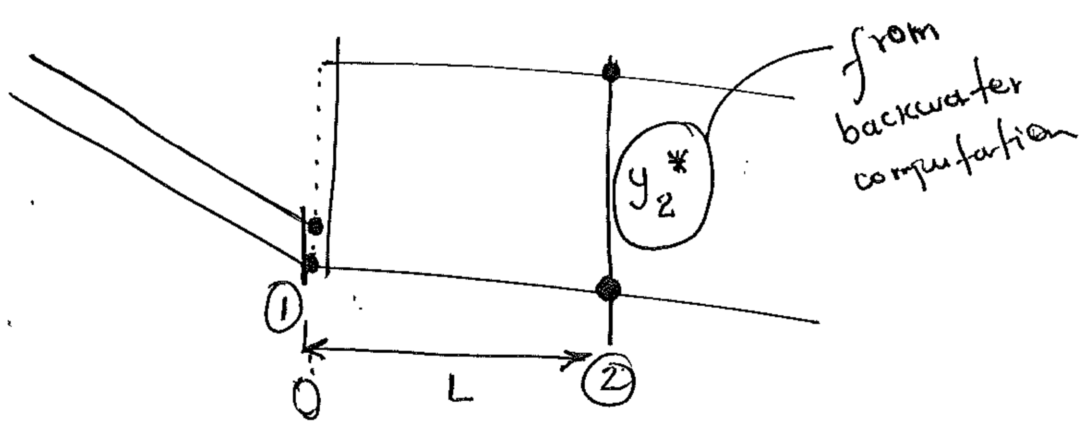
$$\boxed{L \approx 6(y_2 - y_1)}$$

Programming of the Hydraulic jump



$E_1 = y_1 + \frac{V_1^2}{2g} + Z_1$

$E_2 = y_2 + \frac{V_2^2}{2g} + Z_2$



7 08/06/03

Hydhaulic Jump programming (Flow diagram)