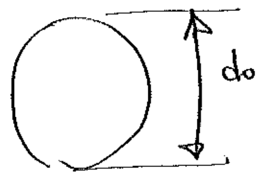
KEIFER AND CHU
BACKWATER PROFILE - CIRCULAR CHANNELS

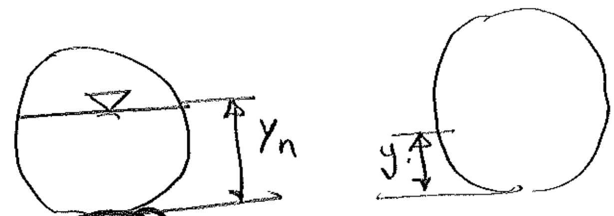
$Q_0 = K_0\sqrt{S_0}$ → flowing full.



$Q = \frac{1}{n} A R^{2/3} S^{1/2}$

$\boxed{K = \frac{1}{n} A R^{2/3}}$

$\frac{K_n}{K_0} = \left(\frac{Q}{Q_0}\right)^2$



$\boxed{Q = K_n\sqrt{S_0}}$

$$\left(\frac{K_n}{K}\right)^2 = \left(\frac{K_n}{K_0}\right)^2\left(\frac{K_0}{K}\right)^2 = \left(\frac{Q}{Q_0}\right)^2\left(\frac{K_0}{K}\right)^2 = \left(\frac{Q}{Q_0}\right)^2 f_1\left(\frac{y}{d_0}\right)$$

$K_0 = f(d_0)$

$K = f(Y)$

$\boxed{Z = \sqrt{A^3/T}}$

$Q = A \cdot V$

$$\alpha\frac{d}{dy}\left(\frac{V^2}{2g}\right) = \alpha\frac{d}{dy}\left(\frac{Q^2}{A^2(2g)}\right) = \frac{\alpha Q^2}{2g}\left(\frac{d}{dy}A^{-2}\right) = \frac{\alpha Q^2}{2g}(-2)A^{-3}\left(\frac{dA}{dy}\right) = -\frac{\alpha Q^2 T}{g A^3}$$

$\longrightarrow \mathbb{F}_r^2$

$$= -\frac{\alpha Q^2}{g Z^2} \quad \cdots (1)$$

Critical flow of discharge equal to Q

$$\frac{\alpha Q^2 T_c}{g A_c^3} = 1$$

$$\frac{\alpha Q^2}{g Z_c^2} = 1 \longrightarrow \boxed{\frac{Q^2}{g} = \frac{Z_c^2}{\alpha}} \quad \cdots (2)$$

$\mathbb{F}^2 = \frac{Q^2}{g\frac{A}{T}A^2}$

$\mathbb{F}^2 = \frac{Q^2 T}{g A^3}$

(2) Into (1)

$$\alpha\frac{d}{dy}\left(\frac{V^2}{2g}\right) = -\frac{\cancel{\alpha}}{\cancel{\alpha}}\left(\frac{Z_c}{Z}\right)^2 \longrightarrow \therefore \boxed{\left(\frac{Z_c}{Z}\right)^2 = \frac{\alpha Q^2 T}{g A^3}} = \frac{\alpha Q^2}{d_0^5}\,\frac{T/d_0}{g(A/d_0^2)^3}$$

$f_2\left(\frac{y}{d_0}\right)$

Gradually varied flow

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 + \alpha d\left(\frac{V^2}{2g}\right)/dy} = \frac{S_0 - S_f}{1 - \alpha\mathbb{F}_r^2}$$

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$$\frac{dy}{dx} = \frac{S_o - S_f}{1 + \alpha\, d\left(V^2/2g\right)/dy} = \frac{S_o - S_f}{1 - \left(Z_c/Z\right)^2} \quad \dots (3)$$ (2)

S_f (Manning's formula) $\quad V = \frac{1}{n} R^{2/3} S_f^{1/2} \quad Q = K\sqrt{S_o} = K\sqrt{S_f}$

$$\boxed{S_f = \frac{Q^2}{K^2}}$$

$$\boxed{S_o = \frac{Q^2}{K_n^2}} \rightarrow \text{(for uniform flow)}$$

$$\boxed{\frac{S_f}{S_o} = \left(\frac{K_n}{K}\right)^2} \rightarrow S_f = \left(\frac{K_n}{K}\right)^2 S_o \quad \dots (4)$$

(4) into (3)

$$\frac{dy}{dx} = \frac{S_o - \left(\frac{K_n}{K}\right)^2 S_o}{1 - \left(Z_c/Z\right)^2} = \frac{S_o\left(1 - \left(\frac{K_n}{K}\right)^2\right)}{1 - \left(\frac{Z_c}{Z}\right)^2}$$

$$\frac{dy}{dx} = \frac{S_o\left[1 - \left(\frac{Q}{Q_o}\right)^2 f_1\left(\frac{y}{d_o}\right)\right]}{1 - \alpha \frac{Q^2}{d_o^5} f_2\left(\frac{y}{d_o}\right)}$$

$$dx = \frac{d_o}{S_o} \frac{\left[1 - \alpha \frac{Q^2}{d_o^5} f_2\left(\frac{y}{d_o}\right)\right]}{1 - \left(\frac{Q}{Q_o}\right)^2 f_1\left(\frac{y}{d_o}\right)} \, d\left(\frac{y}{d_o}\right) \qquad \int_0^X dx = \int_0^{y/d_o} \bigcirc$$

Integrating

$$\boxed{\int_{X_1}^{X_2} X = \frac{d_o}{S_o}\left[\int_0^{\frac{y_1}{d_o}} \frac{d\left(\frac{y}{d_o}\right)}{1 - \left(\frac{Q}{Q_o}\right)^2 f_1\left(\frac{y}{d_o}\right)} - \frac{\alpha Q^2}{d_o^5} \int_0^{\frac{y}{d_o}} \frac{f_2\left(\frac{y}{d_o}\right) d\left(\frac{y}{d_o}\right)}{1 - \left(\frac{Q}{Q_o}\right)^2 f_1\left(\frac{y}{d_o}\right)}\right]_{y_1}^{y_2}}$$

or

$$x = -\frac{d_o}{S_o}\left(X - \frac{\alpha Q^2}{d_o^5} Y\right) + \text{constant.}$$

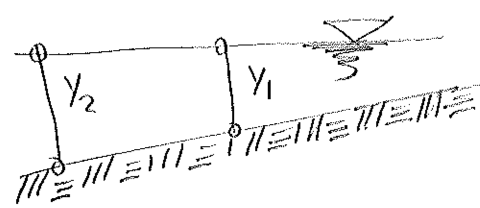
where: $$X = F_1\left(\frac{y}{d_o}, \frac{Q}{Q_o}\right) = \int_0^{\frac{y}{d_o}} \frac{-d(y/d_o)}{1 - (Q/Q_o)^2 f_1(y/d_o)}$$

and

$$Y = F_2\left(\frac{y}{d_o}, \frac{Q}{Q_o}\right) = \int_0^{y/d_o} \frac{-f_2(y/d_o)\, d(y/d_o)}{1 - \left(\frac{Q}{Q_o}\right)^2 f_1(y/d_o)}$$

The length of flow profile between two consecutive sections of depth y_1 and y_2 in a circular conduit may be expressed as:

$$L = A\left[(X_2 - X_1) - B(Y_2 - Y_1)\right]$$



$$\boxed{L = A\left[X_2 - BY_2\right] - A\left[X_1 - BY_1\right]}$$

$A = -d_o/S_o$

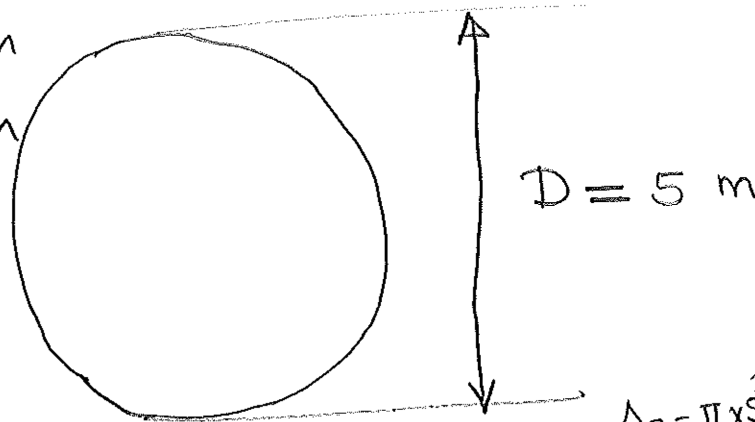
$B = \alpha Q^2/d_o^5$

Numerical computation :

$Q_{actual} = 20\ m^3/s$ $\quad y_1 = 3.50\ m$

$S = 0.001$ $\quad y_2 = 4.00\ m$

$n = 0.015$ $\quad \Delta y = 0.50\ m$



$$\frac{Q}{Q_o} = \frac{20}{48.02} = 0.416$$

$$A_o = \frac{\pi \times 5^2}{4} = 19.63\ m^2$$

$$f_1\left(\frac{y}{d_o}\right) = \left(\frac{K_o}{K}\right)^2$$

$$Q_o = \frac{1}{n} A_o R_o^{2/3} S_o^{1/2}$$

$$R = \frac{D}{4} = 1.25\ m$$

$$K_o^2 = \frac{Q_o^2}{S_o} = \frac{48.02^2}{0.001} = 2{,}305\,920.4$$

$$Q_o = \frac{1}{0.015}(19.63)(1.25)^{2/3} \times (0.001)^{0.5}$$

$$\boxed{Q_o = 48.02\ m^3/s}$$

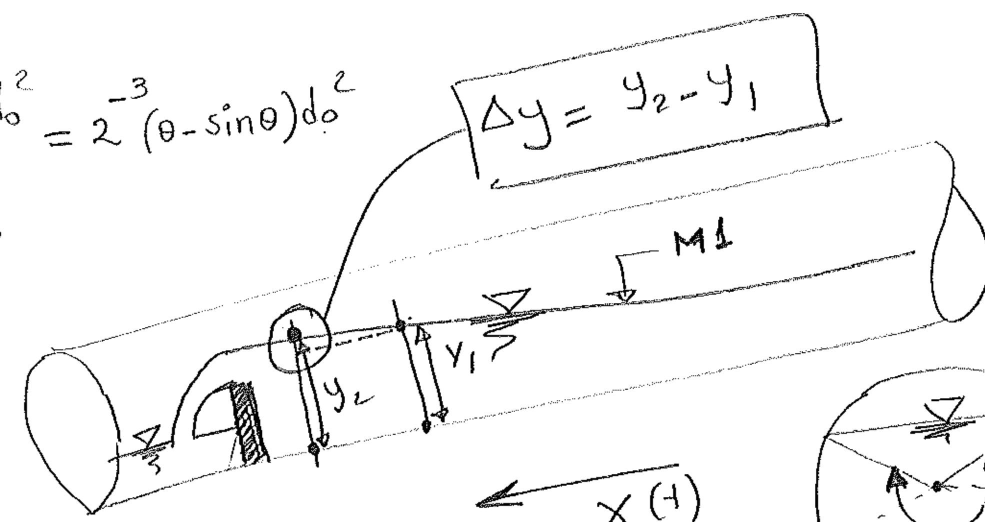
$$K = \frac{1}{n} A R^{2/3} = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}}$$

$\therefore$ $=$

$y_1 = 3.50$

$$A = \frac{1}{8}(\theta - \sin\theta)\, d_o^2 = 2^{-3}(\theta - \sin\theta)\, d_o^2$$

$$P = \frac{1}{2}\theta d_o = 2^{-1}\theta d_o$$

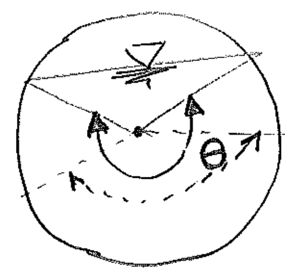


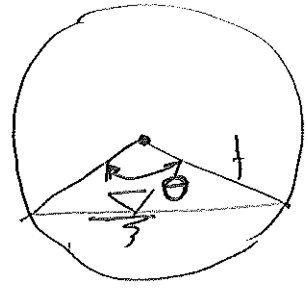
$$\frac{2^{-5}(\theta - \sin\theta)^{5/3}\, \cancel{d_o}^{10/3}}{2^{-2/3}\, \theta^{2/3}\, \cancel{d_o}^{2/3}}$$

$$\frac{2^{-13/3}(\theta - \sin\theta)^{5/3}\, d_o^{8/3}}{\theta^{2/3}}$$

$$2^{\frac{-5+2}{3}}$$

$$= \frac{1}{n}\left[\frac{2^{-13}(\theta - \sin\theta)^5\, d_o^8}{\theta^2}\right]^{1/3}$$



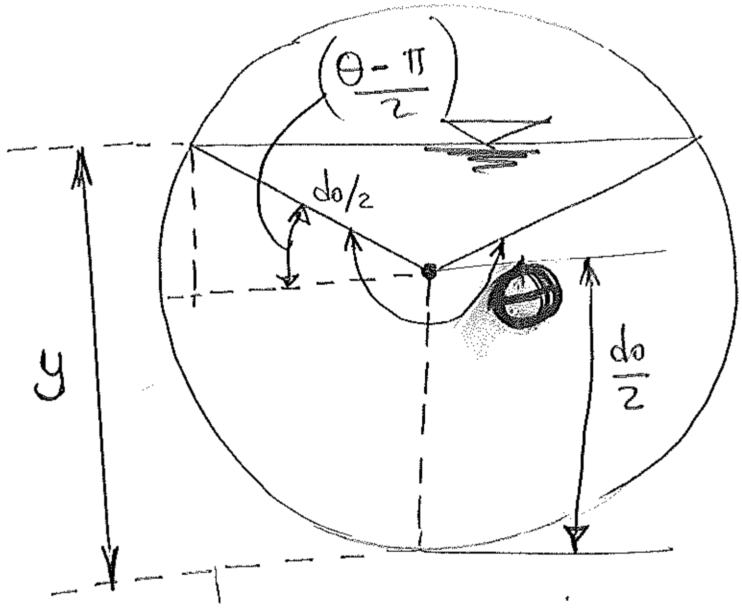


$25 = \frac{1}{0.015}$

"θ"

$$\boxed{\theta = 2\arccos\left(1 - \frac{2y}{d_o}\right)}$$

$$\boxed{K = \frac{1}{n}\left[\frac{2^{-13}(\theta - \sin\theta)^5 d_o^{\,8}}{\theta^2}\right]^{1/3}}$$



$K_{y_1 = 3.50} =$

$K_{y_2 = 4.00} =$

$f_1(y_1/d_o) = \left(\frac{K_o}{K_{y_1}}\right)^2$

$f_1(y_2/d_o) = \left(\frac{K_o}{K_{y_2}}\right)^2$

$$f_2\left(\frac{y}{d_o}\right) = \frac{T/d_o}{g\left(A/d_o^2\right)^3}$$

$$= \frac{\left(\sin\frac{\theta}{2}\right)\cancel{\frac{d_o}{d_o}}}{g\left[\frac{1}{8}(\theta - \sin\theta)\cancel{\frac{d_o^2}{d_o^2}}\right]^3} = \frac{\sin\frac{\theta}{2}}{g \times 2^{-9}(\theta - \sin\theta)^3}$$

$$= \boxed{\frac{2^9 \sin\frac{\theta}{2}}{g(\theta - \sin\theta)^3}}$$

gravity

$$y = \frac{d_o}{2} + \frac{d_o}{2}\sin\left(\frac{\theta}{2} - \frac{\pi}{2}\right) \quad (1)$$

$$y = \frac{d_o}{2} + \frac{d_o}{2}\left[\sin\frac{\theta}{2}\cos\frac{\pi}{2} - \cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\pi}{2}\right)\right]$$

$$\boxed{y = \frac{d_o}{2}\left(1 - \cos\left(\frac{\theta}{2}\right)\right)} \quad \text{when } \theta = 2\pi$$

$$\frac{2y}{d_o} = 1 - \cos\left(\frac{\theta}{2}\right)$$

$$\cos\left(\frac{\theta}{2}\right) = 1 - \frac{2y}{d_o}$$

$$\frac{\theta}{2} = \arccos\left(1 - \frac{2y}{d_o}\right)$$

$$\boxed{\theta = 2\arccos\left(1 - \frac{2y}{d_o}\right)}$$

$$\frac{\theta}{2} = \arccos\left(1 - \frac{2y}{d_o}\right)$$

$f_2(y_1/d_o) =$

$f_2(y_2/d_o) =$

$$\cos\frac{\theta}{2} = \frac{d_o - 2y}{d_o}$$