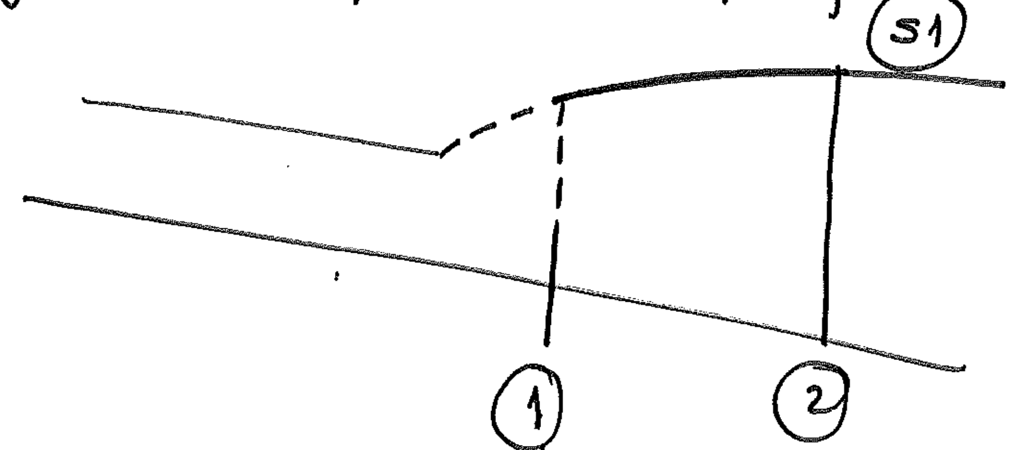
Hydraulic Performance Graph for Steep-slope channel.



$$E_1 = E_2 + h_f$$ $\rightarrow$ known

$$f(y) = E_2 - \underbrace{E_1}_{\text{known}} + \bar{S}_f \underbrace{\Delta x}_{\text{positive}}$$

$$\bar{S}_f = \frac{S_{f1} + S_{f2}}{2}$$

$$E_2 = Y_2 + Z_2 + \frac{V_2^2}{2g}.$$

$$\frac{df}{dy} = \left(1 + 0 + \frac{d}{dy}\left(\frac{V_2^2}{2g}\right)\right) + \frac{d\,S_{f2}}{dy}\ \frac{\Delta x}{2}$$

$$\frac{d}{dy}\left(\frac{V_2^2}{2g}\right) = -\frac{Q^2 T_2}{g A_2^3}$$

$$\frac{d\,S_{f2}}{dy} = \frac{Q^2 n^2}{K_S^2}\ \frac{d}{dy}\left[P^{4/3} A^{-10/3}\right]$$

$\hookrightarrow$ see $\frac{d}{dy}(\ )$ for subcritical flows.

HPG Supercritical flow

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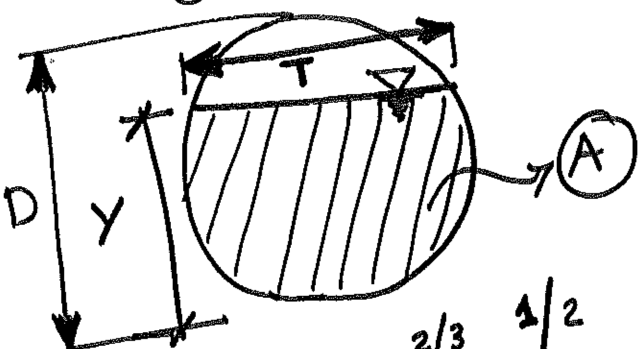
Threshold discharge Q_s

The threshold discharge Q_s is the discharge for which the channel slope is critical.

Q_s $(Y_n = Y_c)$.

$$\frac{Q^2 T}{g A^3} = 1 \quad \ldots ① \quad \text{(critical depth)}$$



① = ②

$$\frac{g\cancel{A^3}^{\,①}}{T} = \frac{k_s^2}{n^2} \cancel{A^2} R^{4/3} S_0$$

$$\frac{R^{4/3}\, T}{A} = \underbrace{\frac{g n^2}{k_s^2 S_0}}_{\text{Known}} \quad \ldots (*)$$

$$Q = \frac{k_s}{n} A R^{2/3} S^{1/2}$$

$$Q^2 = \frac{k_s^2}{n^2} A^2 R^{4/3} S \quad \ldots ②$$

$$f = \frac{R^{4/3}\, T}{A} - \frac{g n^2}{k_s^2 S_0}$$

$A = f(Y, d_o)$

$R = f(Y, d_o)$

$T = f(Y, d_o)$

$$\frac{df}{dy} = \frac{A \dfrac{d(R^{4/3} T)}{dy} - R^{4/3} T \dfrac{dA}{dy}}{A^2}$$

$dA = T\,dy$

$$\frac{dA}{dy} = T$$

$$\frac{df}{dy} = \frac{A\left[R^{4/3}\dfrac{dT}{dy} + T\left(\frac{4}{3}\right)R^{1/3}\dfrac{dR}{dy}\right] - R^{4/3}\, T \overbrace{\dfrac{dA}{dy}}^{T}}{A^2}$$

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$$T = 2\sqrt{y(d_o - y)}$$

$$\boxed{\frac{dT}{dy} = 2\left[y(d_o-y)\right]^{-1/2}(d_o - 2y)}$$

In ✱

$$\frac{R^{4/3}\,T}{A} = \frac{\rho n^2}{k_s^2 S_o} \qquad R = \frac{A}{P}$$

$$\frac{A^{4/3}\,T}{P^{4/3}A} = \frac{\rho n^2}{k_s^2 S_o} \longrightarrow \frac{A^{1/3}}{P^{4/3}}T = \frac{\rho n^2}{k_s^2 S_o}$$

New $$\boxed{f = \frac{A^{1/3}\,T}{P^{4/3}} - \underbrace{\frac{\rho n^2}{k_s^2 S_o}}_{\text{constant}}}$$

$$\frac{df}{dy} = \frac{P^{4/3}\frac{d(A^{1/3}T)}{dy} - A^{1/3}T\left(\frac{4}{3}P^{1/3}\right)\frac{dP}{dy}}{P^{8/3}}$$

$$\frac{df}{dy} = \frac{\frac{P^{4/3}}{P^{1/3}}\left[A^{1/3}\frac{dT}{dy} + T\left(\frac{1}{3}\right)A^{-2/3}\frac{dA}{dy}\right] - A^{+1/3}\,T\left(\frac{4}{3}\cancel{\frac{P^{1/3}}{P^{1/3}}}\right)\frac{dP}{dy}}{P^{8/3}/P^{1/3}}$$

T

$$\frac{dP}{dy} = \frac{2}{\sqrt{1-\left(1-\frac{2y}{d_o}\right)^2}}$$

$$\boxed{\frac{df}{dy} = \frac{P A^{1/3} \frac{dT}{dy} + \frac{1}{3} T^{2} A^{-2/3} - \frac{4}{3} A^{+1/3} T \frac{dP}{dy}}{P^{7/3}}}$$