CWR 3201 Fluid Mechanics, Fall 2018

Fluid Statics

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2.1 INTRODUCTION

**Fluid Statics:** Study of fluids with no relative motion between fluid particles.

- No shearing stress (no velocity gradients)
- Only normal stress exists (pressure)

![Diagram of fluid statics examples](image)

**Fig. 2.1** Examples included in fluid statics: (a) liquids at rest; (b) linear acceleration; (c) angular rotation.
MOTIVATION

Source: asciencem.com, Youtube
(https://www.youtube.com/watch?v=jqpl4ME6rRY)
MOTIVATION (CONT.)

Youtube (https://www.youtube.com/watch?v=Zip9ft1PgV0)
MOTIVATION (CONT.)

Youtube (https://www.youtube.com/watch?v=9jLQx3kD7p8)
2.2 PRESSURE AT A POINT

- Pressure is an infinitesimal normal compressive force divided by the infinitesimal area over which it acts.

- From Newton’s Second Law (for $x$- and $y$-directions):

**Fig. 2.2** Pressure at a point in a fluid.
2.2 PRESSURE AT A POINT

- Pressure in a fluid is **constant** at a point.
- Pressure is a **scalar** function.
- It acts **equally in all directions** at a point for both static and dynamic fluids.

As the element goes to a point \((\Delta x, \Delta y \to 0)\)

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**Fig. 2.2** Pressure at a point in a fluid.
2.3 DERIVATION OF GENERAL FORM OF PRESSURE VARIATION

- Newton’s Second Law in “x”, “y” and “z”-directions:

![Diagram showing forces acting on an infinitesimal element](image)

**Fig. 2.3** Forces acting on an infinitesimal element that is at rest in the xyz-reference frame. The reference frame may be accelerating or rotating.
Using the Chain rule, the pressure change in any direction can be calculated as:

Then the pressure differential becomes:
2.4 FLUIDS AT REST

• The pressure differential (from the previous slide) is:

• At rest, there is no acceleration \((a = 0)\):

No pressure variation in the \(x\)- and \(y\)-directions (horizontal plane). Pressure varies in the \(z\)-direction only (\(dp\) is negative if \(dz\) is positive).

**Pressure decreases as we move up and increases as we move down.**
2.4 FLUIDS AT REST

2.4.1 Pressure in Liquids at Rest

• At a distance $h$ below a free surface, the pressure is:

\[ p = 0 \text{ at } h = 0. \]

Fig. 2.4  Pressure below a free surface.
2.4 FLUIDS AT REST

2.4.3 Manometers
Manometers are instruments that use columns of liquid to measure pressures.

• (a) displays a U-tube manometer used to measure relatively small pressures
• (b): Large pressures can be measured using a liquid with large $\gamma_2$.
• (c): Very small pressures can be measured as small pressure changes in $p_1$, leading to a relatively large deflection $H$.

Fig. 2.7 Manometers: (a) U-tube manometer (small pressures); (b) U-tube manometer (large pressures); (c) micromanometer (very small pressure changes).
Example: P.2.40. Find the gage pressure in the water pipe shown in Fig. P2.40
Example: P.2.41. For the inclined manometer containing mercury, shown in Fig. P2.41, determine the pressure in pipe $B$ if the pressure in pipe $A$ is 10 kPa. Pipe $A$ has water flowing through it, and oil is flowing in pipe $B$. 

![Manometer Diagram](image)
Example: P.2.42. The pressure in pipe B in Problem P2.41 is reduced slightly. Determine the new pressure in pipe B if the pressure in pipe A remains the same and the reading along the inclined leg of the manometer is 11 cm (Tip: See problems 2.41 and 2.42)
2.4 FLUIDS AT REST

2.4.4 Forces on Plane Areas

- The total force of a liquid on a plane surface is:

- After knowing the equation for pressure \((P = \gamma h)\):

![Diagram of force on an inclined plane area](image)
2.4.4 Forces on Plane Areas

\( \bar{h} \): Vertical distance from the free surface to the centroid of the area

\( p_C \): Pressure at the centroid

The centroid or geometric center of a plane figure is the arithmetic mean ("average") position of all the points in the shape.
2.4 FLUIDS AT REST

- The center of pressure is the point where the resultant force acts:
  - Sum of moments of all infinitesimal pressure forces on an area, A, equals the moment of the resultant force.

**Fig. 2.8** Force on an inclined plane area.
\( \bar{y} \): Measured parallel to the plane area to the free surface

- The moments of area can be found using:

See Appendix C for centroids and moments
Fig. 2.9  Force on a plane area with top edge in a free surface.
Example: P.2.56. Determine the force $P$ needed to hold the 4-m wide gate in the position shown in Fig. P2.56.
Example: P.2.62. At what height $H$ will the rigid gate, hinged at a central point as shown in Fig. P2.62, open if $h$ is:

a) 0.6 m? b) 0.8 m? c) 1.0 m?
2.4.5 Forces on Curved Surfaces

https://www.youtube.com/watch?v=zV-JO-l7Mx4

- Direct integration cannot find the force due to the hydrostatic pressure on a curved surface.

- A free-body diagram containing the curved surface and surrounding liquid needs to be identified.
Example: P.2.72. Find the force $P$ required to hold the gate in the position shown in Fig. P.2.72. The gate is 5-m wide.
Example: P.2.77. Find the force $P$ if the parabolic gate shown in Fig. P.2.77 is

a) 2-m wide and $H = 2$ m

b) 4-ft wide and $H = 8$ ft.
2.4 FLUIDS AT REST

2.4.6 Buoyancy (Archimedes’ principle)

https://www.youtube.com/watch?v=2ReflvqaYg8

- Buoyancy force on an object equals the weight of displaced liquid.

Fig. 2.12  Forces on a submerged body: (a) submerged body; (b) free-body diagram; (c) free body showing the buoyant force $F_B$.

$V$ is the volume of displaced fluid and $W$ is the weight of the floating object.
2.4 FLUIDS AT REST

2.4.6 Buoyancy (Archimedes’ principle)

- The buoyant force acts through the centroid of the displaced liquid volume.
- An application of this would be a hydrometer that is used to measure the specific gravity of liquids.
  - For pure water, this is 1.0
2.4 FLUIDS AT REST
2.4.6 Buoyancy (Hydrometers)

Fig. 2.13 Forces on a floating object.

Fig. 2.14 Hydrometer: (a) in water; (b) in an unknown liquid.

• Where $\Delta h$ is the displaced height
• $A$: Cross-sectional area of the stem
• $S_x = \frac{\gamma_x}{\gamma_{water}}$
• For a given hydrometer, $V$ and $A$ are fixed.
Example: P.2.78. The 3-m wide barge shown in Fig. P.2.78 weighs 20 kN empty. It is proposed that it carry a 250-kN load. Predict the draft in:

a) Fresh water
b) Salt water \((S = 1.03)\)
2.4 FLUIDS AT REST

2.4.7 Stability

- In (a) the center of gravity of the body is above the centroid C (center of buoyancy), so a small angular rotation leads to a moment that increases rotation: unstable.
- (b) shows neutral stability as the center of gravity and the centroid coincide.
- In (c), as the center of gravity is below the centroid, a small angular rotation provides a restoring moment and the body is stable.

Fig. 2.15 Stability of a submerged body: (a) unstable; (b) neutral; (c) stable.
2.4 FLUIDS AT REST

Metacentric height

https://www.youtube.com/watch?v=QUgXf2Rj2YQ

Fig. 2.16 Stability of a floating body: (a) equilibrium position; (b) rotated position.

- The **metacentric height** $\overline{GM}$ is the distance from G to the point of intersection of the buoyant force before rotation with the buoyant force after rotation.

- If $\overline{GM}$ is positive: Stable
- If $\overline{GM}$ is negative: Unstable
Fig. 2.17 Uniform cross section of a floating body.
Fig. 2.17 Uniform cross section of a floating body.
Example: P.2.94. The barge shown in Fig. P2.94 is loaded such that the center of gravity of the barge and the load is at the waterline. Is the barge stable?
Example: P.2.92. For the object shown in Fig. P2.92, calculate $S_A$ for neutral stability when submerged.
Linearly Accelerating Containers

Source: asciencemcom, Youtube (https://www.youtube.com/watch?v=jqpl4ME6rRY)
Pressure within an Accelerating Container

Depth of point = 2.9 m  Gage pressure = 28. kPa
Drag the ball to see the pressure change.

Source: Jon Barbieri and Peter Hassinger, "Pressure within an Accelerating Container"
http://demonstrations.wolfram.com/PressureWithinAnAcceleratingContainer/
2.5 LINEARLY ACCELERATING CONTAINERS

• When the fluid is linearly accelerating with horizontal \((a_x)\) and vertical \((a_z)\) components:

![Diagram of linearly accelerating tank](image)

- The derived pressure differential equation is:

**Fig. 2.18** Linearly accelerating tank.
2.5 LINEARLY ACCELERATING CONTAINERS

- As points 1 and 2 lie on a constant-pressure line:

Fig. 2.18 Linearly accelerating tank.

\( \alpha \) = angle that the constant-pressure line makes with the horizontal.
Example: P.2.97. The tank shown in Fig. P2.97 is accelerated to the right at 10 m/s². Find:

a) $P_A$, b) $P_B$, c) $P_C$

Fig. P2.97
Example: P.2.99. The tank shown in Fig. P2.99 is filled with water and accelerated. Find the pressure at point $A$ if $a = 20 \, \text{m/s}^2$ and $L = 1 \, \text{m}$. 

![Diagram of a tank on an inclined plane]
• For a liquid in a rotating container (cross-section shown):

- In a short time, the liquid reaches static equilibrium with respect to the container and the rotating $rz$-reference frame.
- Horizontal rotation will not affect the pressure distribution in the vertical direction.
- No variation in pressure with respect to the $\theta$-coordinate.

**Fig. 2.19** Rotating container: (a) liquid cross section; (b) top view of element.

https://www.youtube.com/watch?v=RdRnB3jz1Yw
2.6 ROTATING CONTAINERS

- Between two points \((r_1,z_1)\) and \((r_2,z_2)\) on a rotating container, the static pressure variation is:
If two points are on a constant-pressure surface (e.g., free surface) with point 1 on the z-axis \([r_1=0]\):

- The free surface is a **paraboloid of revolution**.
Example: P.2.106. For the cylinder shown in Fig. P2.106, determine the pressure at point A for a rotational speed of 5 rad/s.
Example: P.2.107. The hole in the cylinder of Problem P2.106 is closed and the air pressurized to 25kPa. Find the pressure at point A if the rotational speed is 5 rad/s.