Florida International University
CWR 3201 Fluid Mechanics, Fall 2021
Mid-term # 2

Instructor: Arturo S. Leon, Ph.D., P.E., D.WRE

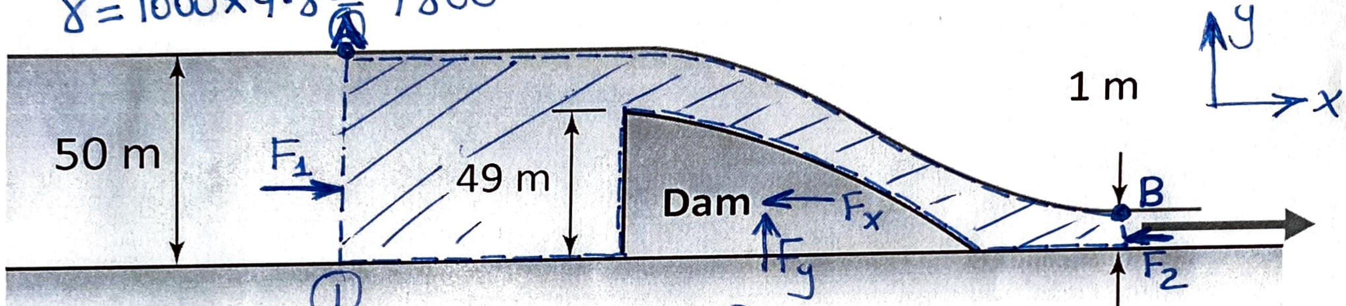
Student Name: Arturo Leon **Panther ID:** ____________

✓ **You will have 50 minutes to complete the exam. The exam is closed book and closed notes. Only one page (front and back) with handwritten equations are allowed**

1. **(35 points) Neglecting viscous effects** and assuming uniform velocity profiles, find the horizontal force component acting on the dam shown in the figure below. The river width is 100 m.

$\rho = 1000\ \text{Kg/m}^3$

$\gamma = 1000 \times 9.8 = 9800$



* Momentum x-direction:

$$F_1 - F_2 - F_x = \dot{m}(V_2 - V_1) \quad \text{...(1)}$$

* Bernoulli equation because of negligible head losses (No viscous effects).

$P_A = P_B$ $\quad V_A^2 \approx 0$

$$\cancel{\frac{P_A}{\gamma}} + \cancel{\frac{V_A^2}{2g}} + Z_A = \cancel{\frac{P_B}{\gamma}} + \frac{V_B^2}{2g} + Z_B$$

$V_A = V_1 = 0$ (Large reservoir compared to section 2)

$$50 = \frac{V_B^2}{2 \times 9.8} + 1 \rightarrow V_B = V_2 = 30.99\ \text{m/s}$$

In (1)

$$1\,225\,000\,000 - 490\,000 - F_x = 100{,}000\,(30.99)(30.99 - 0)$$

$$\boxed{F_x = 1'128{,}472\ \text{KN}}$$

$F_1 = \gamma \bar{h}_1 A_1 = 9800 \times \frac{50}{2} \times 50 \times 100$

$F_1 = 1'225{,}000\ \text{KN}$

$F_2 = \gamma \bar{h}_2 A_2 = 9800 \times \frac{1}{2} \times 1 \times 100$

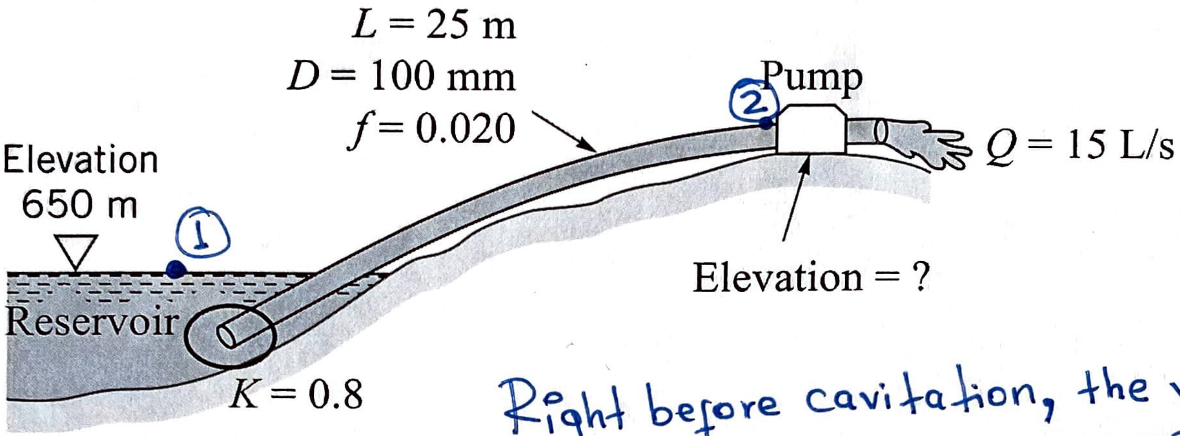
$F_2 = 490\ \text{KN}$

$\dot{m} = \rho A V = \rho A_2 V_2$

$= 1000 \times (1 \times 100)\, V_2$

$= 100{,}000\, V_2$

2. **(30 points)** The 25-m long, 100-mm diameter pipe with a friction factor of 0.020 is used to pump 30°C water from a reservoir as shown below. Determine the maximum elevation of the pump if the flow discharge is 15 L/s right before cavitation. Use P_{vapor} (30°C) = 4.24 kPa, P_{atm} = 101.3 kPa, and water specific weight at 30°C, γ = 9.768 kN/m³.



Right before cavitation, the vapor pressure at ② will be 4.24 kPa ($T = 30°c$).

$Q = 15\ L/S$

$V = \frac{Q}{A} = \frac{15}{1000\,(\pi \times 0.1^2)/4}$

$V = 1.91\ m/s$

* Energy equation between ① and ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + \left(\frac{fL}{D} + \Sigma K\right)\frac{V_2^2}{2g}$$

$\frac{V_1^2}{2g} \approx 0$

$$\frac{101.3 \times 1000}{9.768 \times 1000} + Z_1 = \frac{4.24 \times 10^3}{9.768 \times 10^3} + Z_2 + \frac{1.91^2}{2 \times 9.8}\left(1 + \frac{0.020 \times 25}{0.10} + 0.8\right)$$

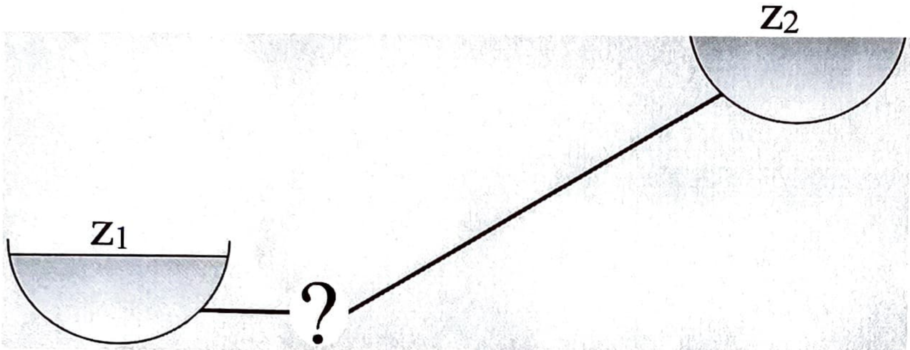
$$\boxed{8.67\ m = Z_2 - Z_1}$$

∴ Pump elevation = 650 + 8.67

= 658.67 m

3- **(35 points)** Water is pumped between two reservoirs in a pipeline with the following characteristics: $D = 300$ mm, $L = 70$ m, $f = 0.025$, $\Sigma K = 2.5$. The radial-flow pump characteristic curve is approximated by the formula $H_P = 30 + 12.7Q - 110Q^2$, where H_P is in meters and Q is in m³/s.

If $z_2 - z_1 = 50$ m, and the minimum required flow discharge is 150 L/s, determine the minimum number of pumps required to meet the minimum flow discharge. Would you use pumps in parallel or in series? Justify your answer.



* Pump curve: $H_p = 30 + 12.7Q - 110Q^2 \quad \cdots ①$

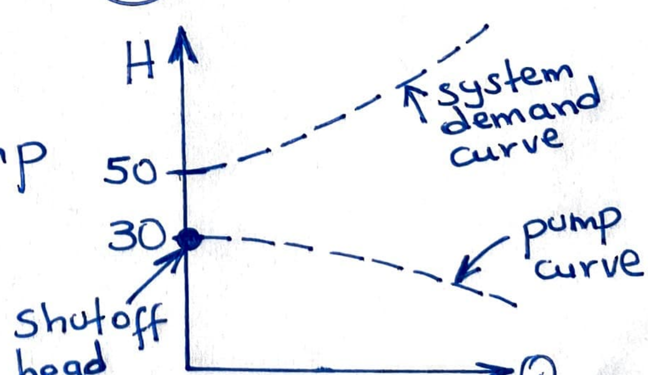
* System demand curve:

$$H_p = Z_2 - Z_1 + \left(\frac{fL}{D} + \Sigma k\right)\frac{Q^2}{2gA^2}$$

$$H_p = 50 + \left(\frac{0.025 \times 70}{0.3} + 2.5\right)\frac{Q^2}{2 \times 9.8 \times \left[\frac{\pi \times 0.3^2}{4}\right]^2}$$

$$H_p = 50 + 85.09\,Q^2 \quad \cdots ②$$

* Because $Z_2 - Z_1$ (50 m) is greater than the single pump shutoff head (30 m), we need to use at least two pumps in series. Pumps in parallel will not provide a head above 30 m.



* We need to use at least two pumps in series and then verify if the flow discharge is above the minimum required flow discharge (150 L/S)

* Let's try with two pumps in series

Combined pump curve

$$H_p = 2(30 + 12.7Q - 110Q^2)$$

$$H_p = 60 + 25.4Q - 220Q^2 \quad \ldots (3)$$

(2) = (3)

$$\therefore \; 50 + 85.09Q^2 = 60 + 25.4Q - 220Q^2$$

$$305.09Q^2 - 25.4Q - 10 = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$Q = \frac{-(-25.4) \pm \sqrt{25.4^2 - 4(305.09)(-10)}}{2(305.09)}$$

$a = 305.09$

$b = -25.4$

$c = -10$

$$Q = \frac{25.4 \pm 113.35}{2(305.09)}$$

$$Q = \begin{cases} 0.227 \; m^3/s \; (227 \; L/s) \\ -0.14 \; (\text{Not possible}) \end{cases}$$

The flow (227 L/s) is higher than the minimum required flow discharge (150 L/s).

Thus, we need two pumps in series.