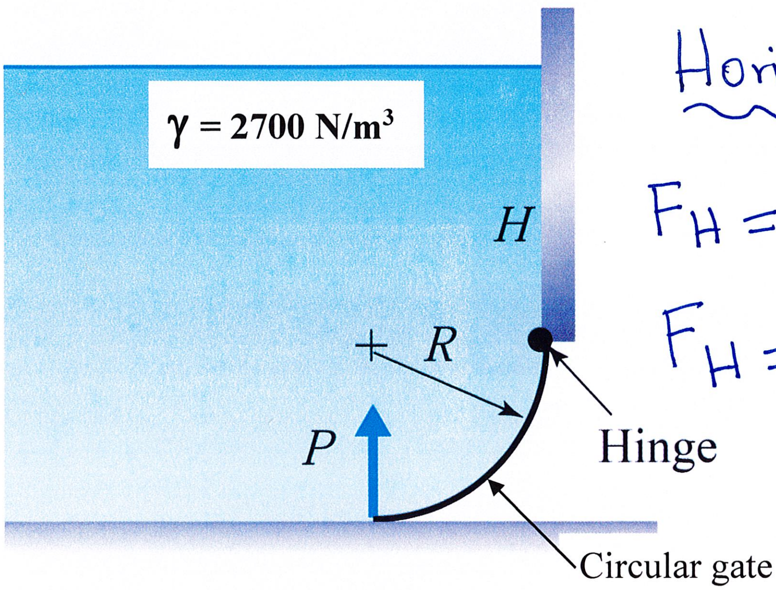
Florida International University
CWR 3201 Fluid Mechanics, Fall 2020
Mid-term

Instructor: Arturo S. Leon, Ph.D., P.E., D.WRE

Student Name: ____ **Date:** 10/16/2020

- ✓ You will have 1h 40 minutes to complete the exam starting at 9:30 am. You will have an extra of 10 minutes to scan your solution and upload it to Canvas [**Assignment** "**Upload your Mid-term Exam Solution HERE**"]. **Canvas will not accept uploads after 11:20 am.**
- ✓ The exam is closed book and closed notes. You can use the one-page formula sheet provided via Canvas. Only one page (front and back) with handwritten equations are allowed.

1. **(25 points)** What is the horizontal and vertical force of the liquid acting on the circular gate below? Use $R = 1.5$ m, $H = 6$ m, Gate width $b = 3$ m. The **liquid has a specific weight of 2700 N/m³**.



Horizontal Force

$$F_H = \gamma \bar{h} A = 2700\left[6+\frac{1.5}{2}\right]\left[1.5 \times 3\right]$$

$$F_H = 82{,}013\ \text{N}$$

$$\boxed{F_H = 82\ \text{kN}}$$

Vertical Force

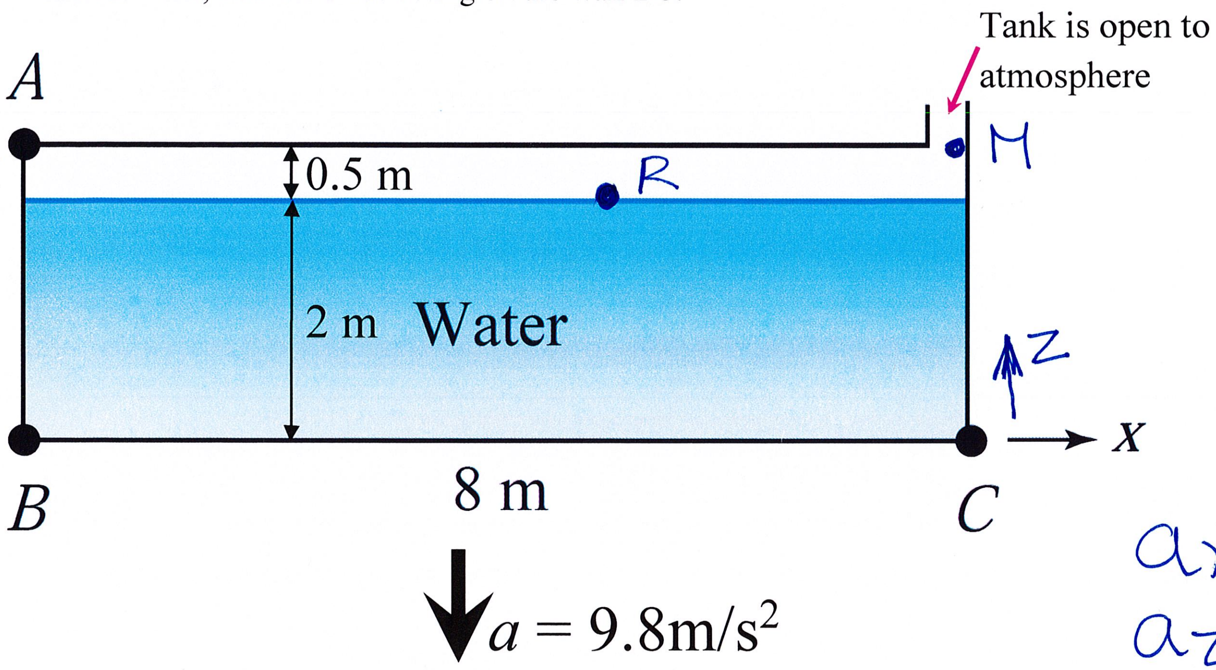
F_V = weight of water over gate

$$F_V = \gamma \forall = 2700 \times 3 \left[6 \times 1.5 + \frac{\pi \times 1.5^2}{4}\right]$$

gate width

$$F_V = 87{,}214\ \text{N} = \boxed{87.2\ \text{kN}}$$

2. **(25 points)** The tank shown in the figure below is accelerated vertically **down** at 9.8 m/s^2. If the tank is 4 meters wide, find the force acting on the wall BC.



$a_x = 0$

$a_z = -9.8 \frac{m}{s^2}$

Solution

$$dp = -\rho \cancel{a_x}^{0} dx - \rho(a_z + g)\,dz$$

$$dp = -0 - \rho(-9.8 + 9.8)\,dz$$

$$\boxed{dp = 0}$$

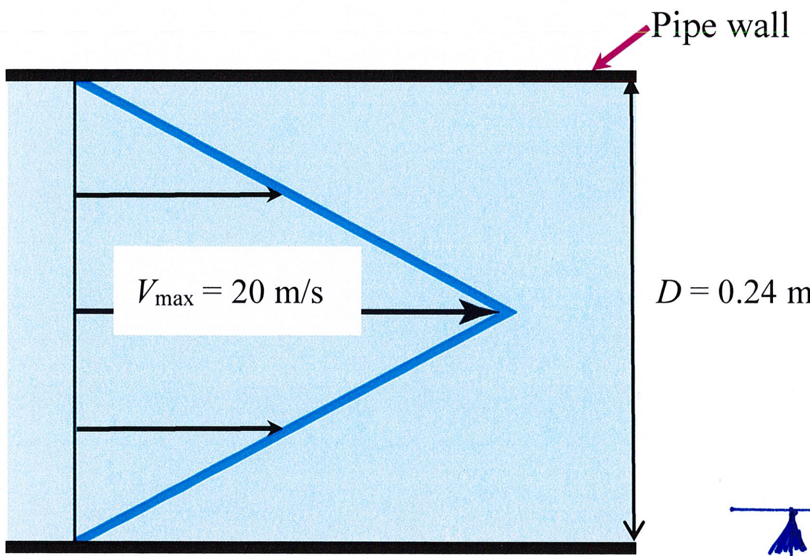
when $a_z = -g$, the particles of water are "floating". There is no pressure difference between them ($dp = 0$).

$$P_R = P_C = P_B = P_A = P_M = 0$$

$$F_{wall\ BC} = P_B \times Area = 0 \times (8 \times 4)$$

$$\boxed{F_{wall\ BC} = 0\ N}$$

3. (**25 points**) Water flows in a 24-cm-diameter pipe with the velocity profile shown below (i.e., the velocity increases linearly from zero at the pipe wall to a maximum at the center of the pipe). Find the flow discharge if the maximum velocity at the center of the pipe (V_{max}) is 20 m/s.



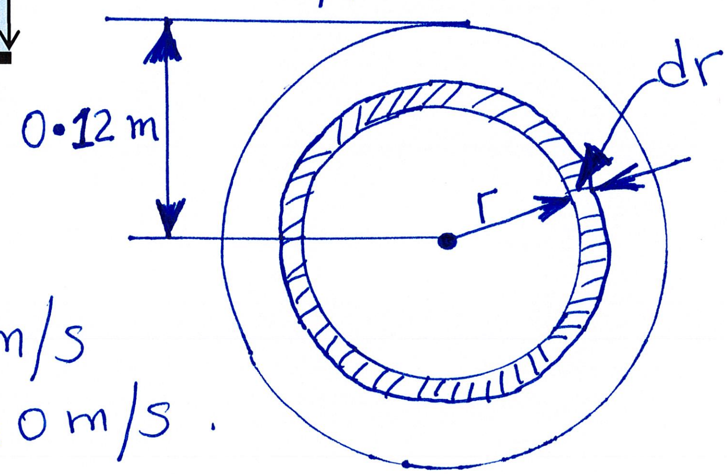
Solution

$$r = \frac{D}{2} = \frac{0.24}{2} = 0.12 \text{ m}$$

$$Q = AV$$

$$Q = \int v \, dA \quad \cdots (1)$$

$$dA = 2\pi r \, dr$$



* Velocity as a function of "r."

V at $r = 0$, $V = 20$ m/s

V at $r = 0.12$ m, $V = 0$ m/s.

line equation

$y = mx + b$ $\qquad V = mr + b$

At $r = 0$: $20 = m(0) + b \rightarrow b = 20$

At $r = 0.12$: $0 = m(0.12) + 20$

$m = -166.67$

$$\therefore \boxed{V = -166.67\, r + 20}$$

In (1) $$Q = \int_0^{0.12} (-166.67 r + 20)(2\pi r)\, dr$$

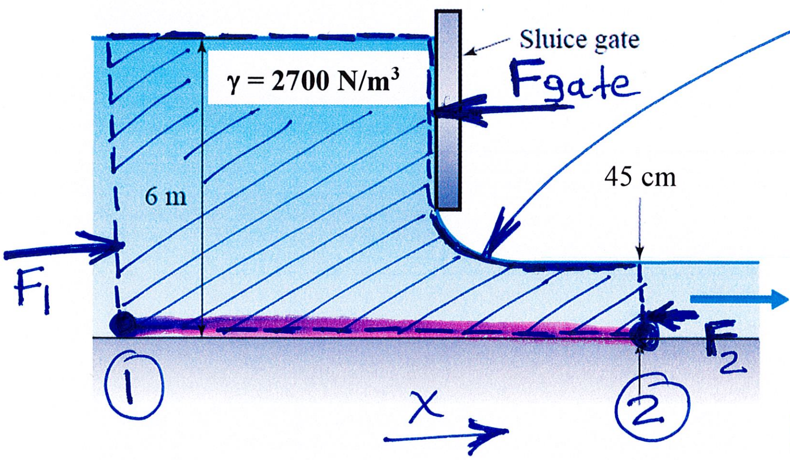
$$Q = 2\pi \left(-166.67 \frac{r^3}{3} + 20 \frac{r^2}{2} \right) \Bigg|_0^{0.12}$$

$$Q = 2\pi \left(-0.096 + 0.144 \right)$$

$$Q = 0.302 \ m^3/s$$

4. **(25 points)** Assuming hydrostatic pressure distributions ($P = \gamma h$), uniform velocity profiles (i.e., velocity doesn't change with depth), and negligible viscous effects, find the horizontal force needed to hold the sluice gate in the position shown below. **The liquid has a specific weight of 2700 N/m³**. The gate width is 5 m.

control volume



Momentum Eq:

$F_1 - F_2 - F_{gate} = \dot{m}(V_2 - V_1)$ ...(1)

$F_1 = \gamma \bar{h}_1 A_1 = 2700\left(\frac{6}{2}\right)(6\times5)$

$F_1 = 243{,}000\ N$

$F_2 = \gamma \bar{h}_2 A_2 = 2700\left(\frac{0.45}{2}\right)(0.45\times5)$

$F_2 = 1367\ N$

* Bernoulli Eq [Remember this applies along a streamline].

Streamline at bottom of channel ($z = z_1 = z_2$)

$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$

$6 + \frac{V_1^2}{2g} + \cancel{z_1} = 0.45 + \frac{V_2^2}{2g} + \cancel{z_2}$ ($z_1 = z_2$)

$\boxed{6 + \frac{V_1^2}{2g} = 0.45 + \frac{V_2^2}{2g}}$ ...(2)

Continuity

$Q_1 = Q_2$

$V_1(6)(\cancel{b}) = V_2(0.45)\cancel{b}$

$\boxed{V_2 = 13.33\ V_1}$

In 2

$$6 + \frac{V_1^2}{2g} = 0.45 + \frac{(13.33\, V_1)^2}{2g}$$

$$5.55 = 176.69 \frac{V_1^2}{19.6} \longrightarrow V_1 = 0.78 \text{ m/s}$$

$$V_2 = 10.46 \text{ m/s}$$

$$* \quad \dot{m} = \rho A V = \rho A_1 V_1 = \rho A_2 V_2$$

$$\dot{m} = \frac{2700}{9.8} \times (6 \times 5)(0.78) = 6446.94 \ \frac{\text{Kg}}{\text{s}}$$

In 1

$$243{,}000 - 1367 - F_{gate} = 6446.94\,(10.46 - 0.78)$$

$$F_{gate} = 179{,}227 \text{ N}$$

$$\boxed{F_{gate} = 179.2 \text{ kN}}$$