Florida International University
CWR 3201 Fluid Mechanics, Fall 2021
Final Exam

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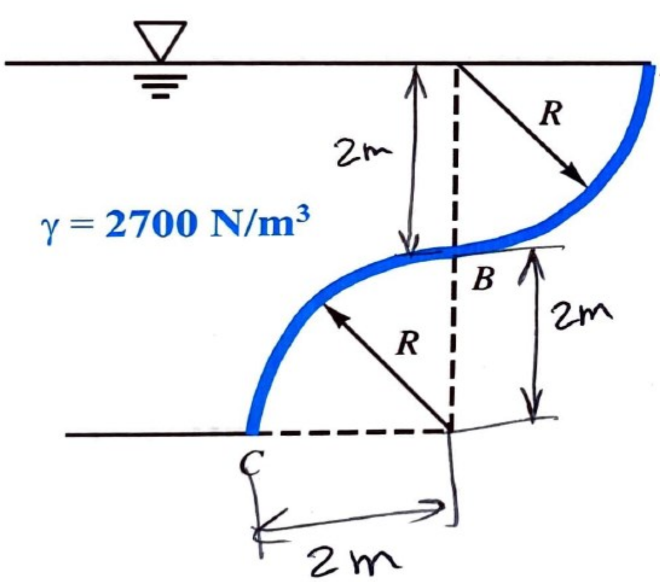
Student Name: ________________________________ **Panther ID:** _________________

- ✓ **You will have 2 hours to complete the exam. The exam is closed book and closed notes**
- ✓ **Only one page (front and back) with handwritten equations are allowed (no photocopies or artificially reduced text will be allowed)**
- ✓ **No cell phones or any type of communication device will be allowed.**
- ✓ **The final exam consists of five questions; however, the grading will be based on four questions only. If five problems are solved, the grading will consider the 4 solutions with the highest scores.**

1. **(25 points)** Calculate the horizontal and vertical forces of the liquid (γ = **2700 N/m^3**) acting on the curved gate *ABC.* Assume R = 2 m and the gate width is 5 m.



$F_H = \gamma \bar{h} A \quad \bar{h} = 2m$

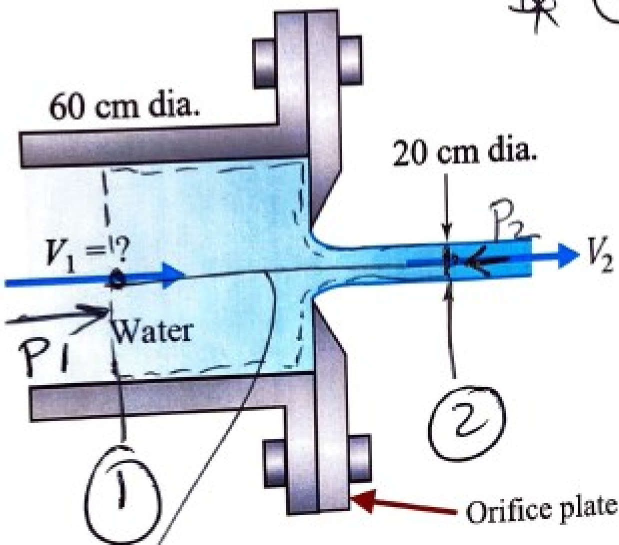
$F_H = 2700 \frac{N}{m^3} * 2 * (4 \times 5)$

$F_H = 108,000\ N$

$\boxed{F_H = 108\ KN}$

$* \ F_V = \gamma \forall = \gamma (2 \times 4) \times 5 = 108,000\ N$

2. **(25 points)** What is the water velocity V_1 inside the pipe if the force needed to hold the orifice plate shown in the figure below is 50,000 N? **Hint:** The orifice plate is circular.



* Continuity

$$Q_1 = Q_2$$

$$V_1 \frac{\pi (0.6)^2}{4} = V_2 \frac{\pi (0.2)^2}{4}$$

$$\boxed{V_2 = 9 V_1} \quad \cdots (1)$$

* Bernoulli

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{\cancel{P_2}^{\,0}}{\gamma} + \frac{V_2^2}{2g} + \cancel{z_2}$$

streamline

$$\frac{P_1}{\gamma} = \frac{V_2^2 - V_1^2}{2g} \rightarrow P_1 = \rho \frac{(V_2^2 - V_1^2)}{2} \quad \cdots (2)$$

* Momentum equation

$$P_1 A_1 - F - \cancel{P_2 A_2}^{\,0} = \dot{m} (V_{2x} - V_{1x})$$

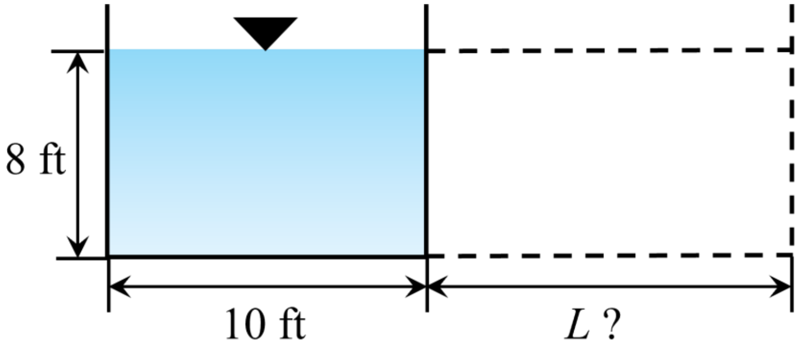
$$\rho \frac{(V_2^2 - V_1^2)}{2} \left[\pi \times \frac{0.6^2}{4} \right] - 50{,}000 = 1000\, V_1\, \pi \times \frac{0.6^2}{4} \left[9V_1 - V_1 \right]$$

$$1000 \left[\frac{81 V_1^2 - V_1^2}{2} \right] \left[\pi \times \frac{0.6^2}{4} \right] - 50{,}000 = 1000\, V_1\, \pi \times \frac{0.6^2}{4} (8 V_1)$$

Solving for V_1.

$$V_1 = 2.35 \text{ m/s}$$

3. **(25 points)** The rectangular canal shown in the figure below is to be widened so that it can carry twice the amount of water. Determine the additional width, ***L***, required if all other parameters (i.e., flow depth, bottom slope, surface material) are to remain the same.



$$\frac{Q_{original}\ (o)}{Q_{widened}\ (w)} = \frac{1}{2}$$

$$(S_o = S_w)$$

$$\frac{Q_o}{Q_w} = \frac{\cancel{\frac{1.49}{n}} A_o R_o^{2/3} \cancel{S_o^{1/2}}}{\cancel{\frac{1.49}{n}} A_w R_w^{2/3} \cancel{S_w^{1/2}}}$$

$$R = \frac{A}{P}$$

$$A_o = 8 \times 10 = 80$$

$$A_w = 80 + 8L$$

$$P_o = 8 \times 2 + 10 = 26$$

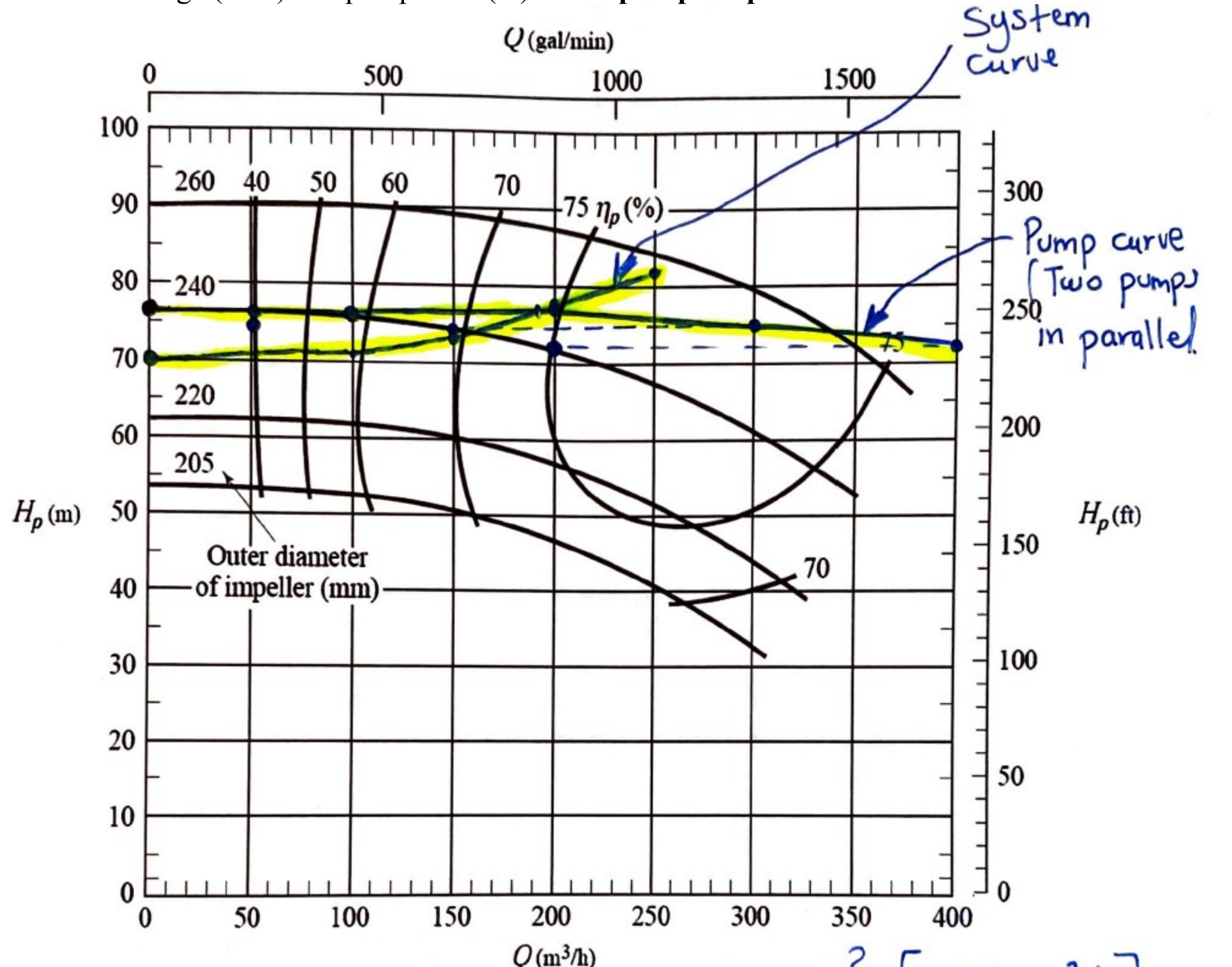
$$P_w = 26 + L$$

$$\frac{1}{2} = \frac{\dfrac{A_o^{5/3}}{P_o^{2/3}}}{\dfrac{A_w^{5/3}}{P_w^{2/3}}}$$

$$\frac{1}{2} = \frac{80^{5/3} * (26+L)^{2/3}}{26^{2/3} * (80+8L)^{5/3}}$$

Solving: $\boxed{L = 6.59\ ft}$

4. **(25 points)** The **240**-mm-diameter pump represented in the figure below is used to move **water** in a piping system. The demand or system curve is $H_p(\text{m}) = 70 + 2500Q^2$, where ***Q* is in cubic meters per second**. Find the flow discharge (m^3/s) and pump head (m) if **two pumps in parallel** are used.



System curve : $H_p(\text{m}) = 70 + 2500\,Q^2 \quad [Q \text{ in } \text{m}^3/\text{s}] \quad \cdots (1)$

$Q\,(\text{m}^3/\text{h})$	$Q\,(\text{m}^3/\text{s})$	$H_p(\text{m})$ [Eq. (1)]
50	0.01389	70.48
100	0.02778	71.93
150	0.04167	74.34
200	0.05556	77.72
250	0.06944	82.06

Plot $Q\,(\text{m}^3/\text{h})$ vs $H_p\,(\text{m})$ (system curve)

$Q \sim 190\ \text{m}^3/\text{h}$

$Q \approx 0.0528\ \text{m}^3/\text{s}$

$Q \approx 52.8\ \dfrac{\text{Liters}}{\text{second}}$

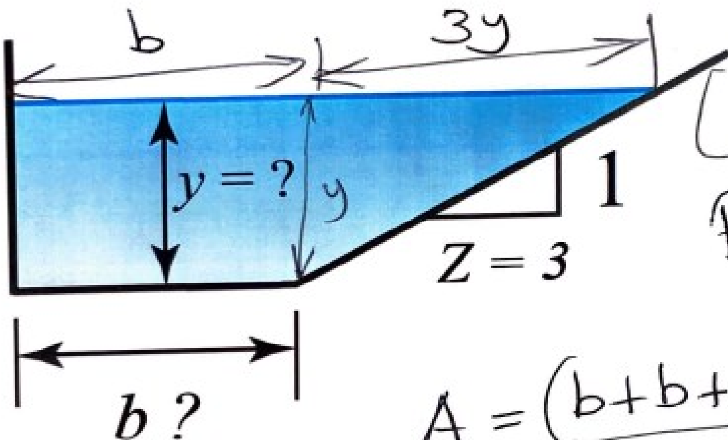
$\boxed{H_p \sim 77\ \text{m}}$

5. **(25 points)** The channel below carries a discharge of 30 m^3/s of water with a velocity of 1.5 m/s. If the channel is designed for **maximum hydraulic efficiency** conditions, what should be the bottom (***b***) and the height (***y***) of the channel?

Derivative rule for a power function: $\frac{dx^n}{dm} = nx^{n-1}\frac{dx}{dm}$

$Q = 30\ m^3/s$

$V = 1.5\ m/s$



$\boxed{A = 20\ m^2}$

$P_{minimum} = \checkmark$

$A = \left(\frac{b+b+3y}{2}\right)y = \left(b + \frac{3}{2}y\right)y$

$\boxed{P = y + b + \sqrt{10}\,y}$

$\frac{dP}{dy} = 0$

$0 = 1 + \frac{db}{dy} + \sqrt{10} \rightarrow \frac{db}{dy} = -1 - \sqrt{10}$

$\boxed{\frac{db}{dy} = -4.16}$

$\frac{dA}{dy} = 0$

$0 = \frac{d}{dy}(by) + \frac{3}{2}\frac{d}{dy}(y^2)$

$b + y\frac{db}{dy} + \frac{3}{\cancel{2}}(\cancel{2}y) = 0$

$$b + y(-4.16) + 3y = 0$$

$$b - 1.16y = 0$$

$$\boxed{b = 1.16y}$$

$$A = 20 = \left(b + \frac{3}{2}y\right)y$$

$$20 = (1.16y + 1.5y)y$$

$$\boxed{20 = 2.66y^2} \rightarrow \boxed{y = 2.74\,m}$$

$$\boxed{b = 3.18\,m}$$