Florida International University
CWR 3201 Fluid Mechanics, Fall 2019
Final Exam

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Student Name: Arturo Leon

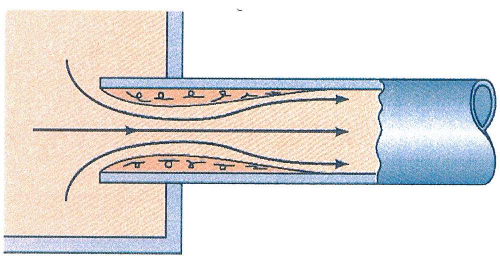
✓ **You will have 2h minutes to complete the exam. The exam is closed book and closed notes**

✓ **Only one page (front and back) with handwritten equations are allowed (no photocopies or artificially reduced text will be allowed)**

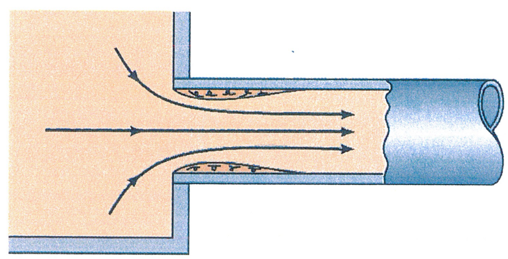
✓ **No cell phones or any type of communication device will be allowed.**

1. **(5 points)** Four different entrance flow conditions are presented below. Which one of the five options correctly lists the local loss coefficient (K) of the four entrance flow conditions **from the largest to smallest**?
    a. (b)-(c)-(d)-(e)
    b. (a)-(b)-(c)-(d)
    c. (d)-(b)-(c)-(a)
    d. (d)-(c)-(b)-(a)
    e. (a)-(c)-(d)-(b)

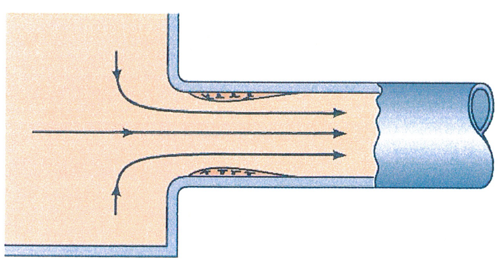
5 points (b)



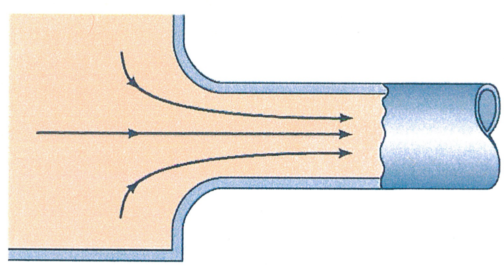
(*a*)



(*b*)

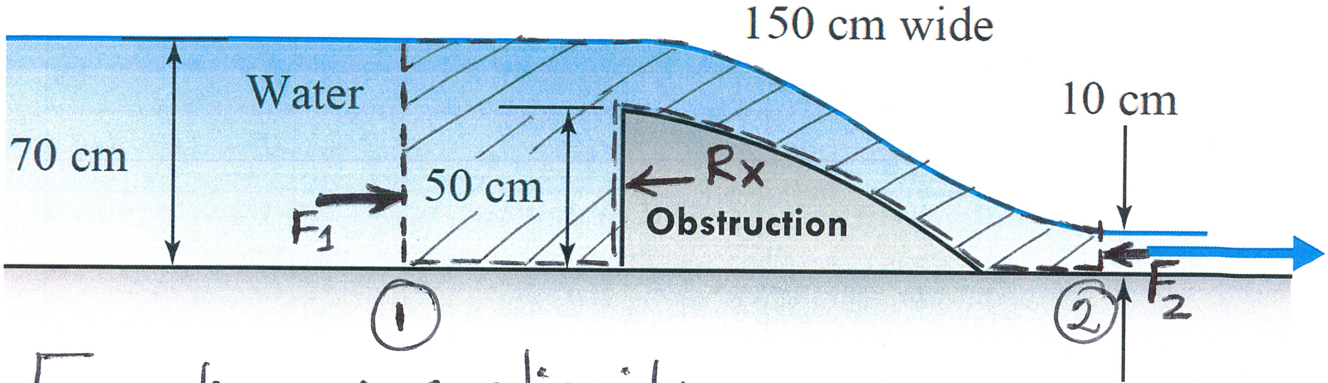


(*c*)



(*d*)

2. (20 points) **Neglecting viscous effects** and assuming uniform velocity profiles, find the horizontal force component acting on the obstruction shown in the figure below. Note that the channel width is 150 cm.



Equation of continuity

$$V_1 (0.7) \cancel{b} = V_2 (0.1) \cancel{b}$$

$$V_2 = 7 V_1 \quad \ldots (1)$$

Equation of Energy: $E_1 = E_2$

$$y_1 + \frac{V_1^2}{2g} + Z_1 = y_2 + \frac{V_2^2}{2g} + Z_2 \quad \ldots (2)$$

From (1)

$$0.7 + \frac{V_1^2}{2g} + 0 = 0.1 + \frac{(7V_1)^2}{2g} + 0$$

$$\frac{48 V_1^2}{2 \times 9.81} = 0.6$$

$$V_1 = 0.495 \text{ m/s} \qquad V_2 = 3.467 \text{ m/s}$$

Momentum: $\Sigma F_x = 0$

$$F_1 - F_2 - R_x = \dot{m}(V_2 - V_1) \quad \cdots (3)$$

$$F_1 = \gamma \bar{h}_1 A = 9810 \times \frac{0.7}{2}(0.7 \times 1.5) = 3605.2 \text{ N}$$

$$F_2 = \gamma \bar{h}_2 A = 9810 \times \frac{0.1}{2}(0.1 \times 1.5) = 73.6 \text{ N}$$

$$\dot{m} = \rho A V = 1000\,(0.7 \times 1.5)(0.495) = 519.75$$

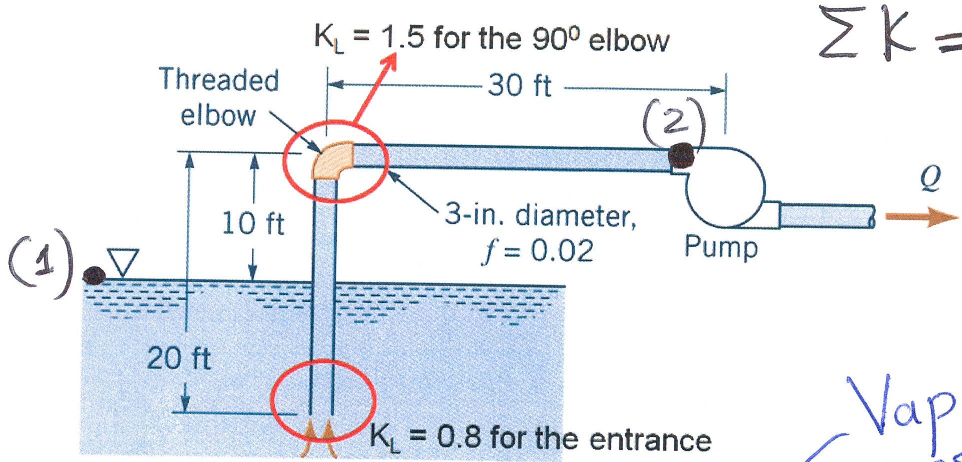
In (3)

$$3605.2 - 73.6 - R_x = 519.75(3.467 - 0.495)$$

$$\boxed{R_x = 1986.9 \text{ N}}$$

Force acting on the obstruction.

3. **(20 points)** Water at 40°F is pumped from a lake as shown in the figure below. The atmospheric pressure is 14.7 psi (absolute) and the vapor pressure at 40°F is 0.127 psi (absolute). Consider that major friction coefficient $f = 0.02$, $K = 1.5$ for the 90^0 elbow and $K = 0.8$ for the pipe entrance. What is the maximum flowrate possible without cavitation occurring? Assume that the specific weight of water is $\gamma_w = 62.4$ lb/ft^3



$\Sigma K = 0.8 + 1.5 = 2.3$

Vapor pressure

* Energy equation $\quad E_1 = E_2$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + \left(\frac{fL}{D} + \Sigma K\right)\frac{V_2^2}{2g}$$

$$\frac{\left(14.7\,\frac{lb}{in^2}\right)\left(144\,\frac{in^2}{ft^2}\right)}{62.4\,\frac{lb}{ft^3}} + 0 = \frac{0.127(144)}{62.4} + \frac{V_2^2}{2\times 32.2} + 10 + \left[\frac{0.02(50)}{(3/12)} + 2.3\right]\frac{V_2^2}{64.4}$$

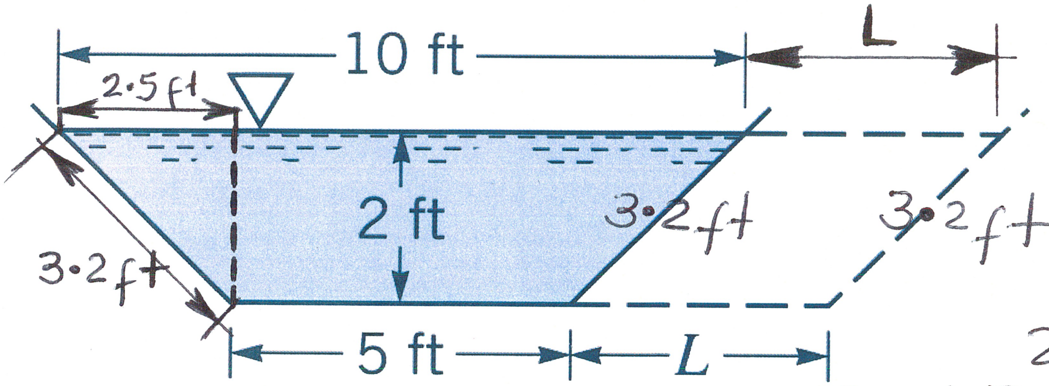
$$\boxed{V_2 = 14.44\,\frac{ft}{s}}$$

$\therefore\ Q = AV$

$$Q = \frac{\pi (3/12)^2}{4} \times 14.44$$

$$\boxed{Q = 0.71\ ft^3/s}$$

4. **(20 points)** The canal shown in the figure below is to be widened so that it can carry twice the amount of water. Determine the additional width, ***L***, required if all other parameters (i.e., flow depth, bottom slope, surface material, side slope) are to remain the same.



Manning's equation: $Q = \frac{C_1}{n} A R^{2/3} S_o^{1/2}$ ... (1)

$$\frac{\text{Original canal (o)}}{\text{widened canal (w)}} = \frac{\cancel{Q}}{2\cancel{Q}} = \frac{\cancel{\frac{C_1}{n}} A_o R_o^{2/3} \cancel{S_o^{1/2}}}{\cancel{\frac{C_1}{n}} A_w R_w^{2/3} \cancel{S_o^{1/2}}}$$

$$\frac{1}{2} = \frac{A_o R_o^{2/3}}{A_w R_w^{2/3}} \quad \ldots (2)$$

$$\frac{1}{2} = \frac{15 \times 1.316^{2/3}}{(15+2L)\left(\frac{15+2L}{11.4+L}\right)^{2/3}}$$

$$\boxed{L = 5.94\ ft}$$

Original (o)

$A_o = \left(\frac{10+5}{\cancel{2}}\right)\cancel{2} = 15\ ft^2$

$P_o = 11.40\ ft$

$R_o = 1.316$

Widened (w)

$A_w = \left(\frac{15+2L}{\cancel{2}}\right)\cancel{2}$

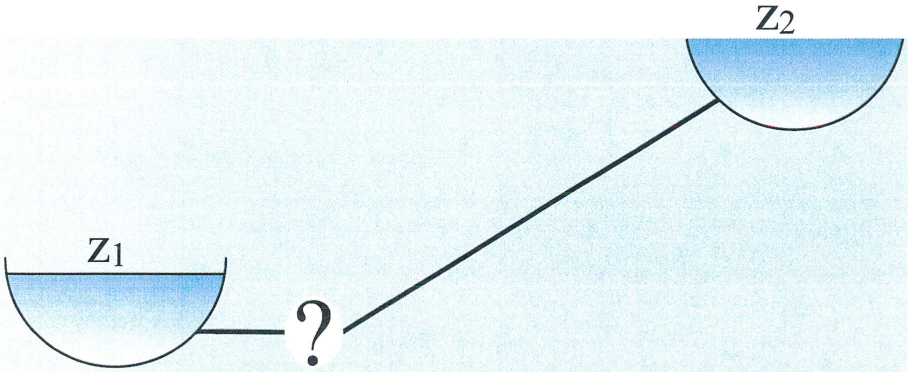
$A_w = 15 + 2L$

$P_w = 11.4 + L$

$R_w = \frac{15+2L}{11.4+L}$

5. **(20 points)** Water is pumped between two reservoirs in a pipeline with the following characteristics: $D = 300$ mm, $L = 70$ m, $f = 0.025$, $\Sigma K = 2.5$. The radial-flow pump characteristic curve is approximated by the formula $H_P = 22.9 + 10.7Q - 111Q^2$, where H_P is in meters and Q is in m³/s.

If $z_2 - z_1 = 74$ m, and the minimum required flow discharge is 0.20 m³/s, determine the minimum number of pumps you would use. Would you use pumps in parallel or in series? Justify your answer.



The system demand curve is:

$$H_p = Z_2 - Z_1 + \left(\frac{fL}{D} + \Sigma K\right)\frac{Q^2}{2gA^2}$$

$$H_p = 74 + \left(\frac{0.025 \times 70}{0.3} + 2.5\right)\frac{Q^2}{2 \times 9.81\left(\frac{\pi \times 0.3^2}{4}\right)^2}$$

$$H_p = 74 + 85\,Q^2 \quad \ldots (1)$$

* Note that $Z_2 - Z_1$ is greater than the single pump shutoff head ($74\text{ m} > 22.9\text{ m}$)

We need to use at least 4 pumps in series and then verify if the flow discharge is above the minimum required flow discharge (0.2 m³/s)

* Assuming 4 pumps in series

Combined pump curve

$$H_p = 4\left(22.9 + 10.7Q - 111Q^2\right) \quad \ldots (2)$$

Equating the combined pump curve and system demand curve.

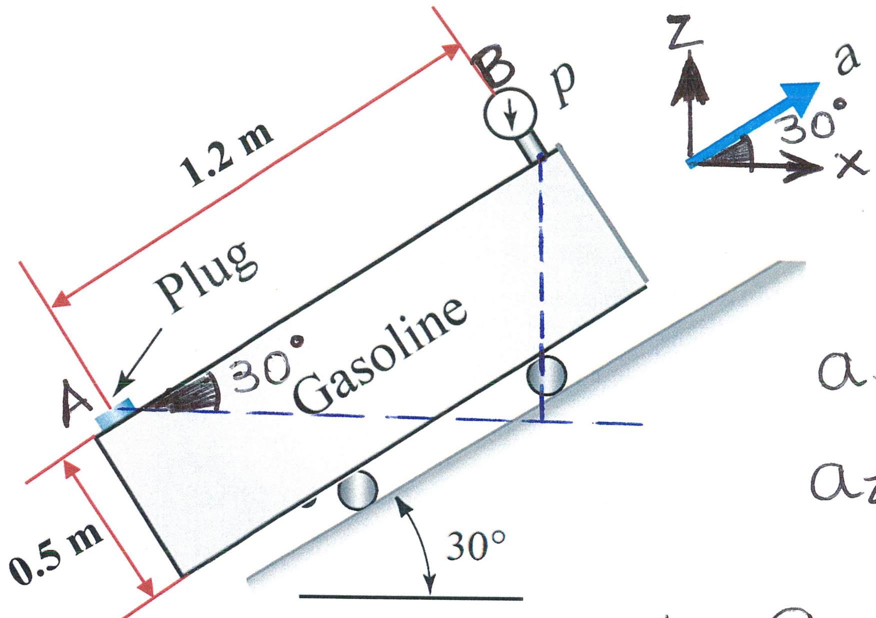
$$(1) = (2) \quad 74 + 85Q^2 = 91.6 + 42.8Q - 444Q^2$$

$$529Q^2 - 42.8Q - 17.6 = 0$$

$$Q = 0.227 \ m^3/s$$

Because $0.227 > 0.2$, only four pumps in series ~~are~~ required.

6. **(15 points)** The initial pressure in the gasoline tank below right before acceleration is $p = 20$ kPa. What is the force of the gasoline acting on the 4-cm-diameter plug when the tank is accelerated as at the rate of $\mathbf{a} = 5$ m/s^2. The density of gasoline is 680 kg/m^3 ($S = 0.68$).



Solution

$$dp = -\rho a_x\,dx - \rho(a_z + g)\,dz$$

$$a_x = 5\cos 30^\circ = 4.33\ \mathrm{m/s^2}$$

$$a_z = 5\sin 30^\circ = 2.5\ \mathrm{m/s^2}$$

Between points A and B

$$P_A - P_B = -S\rho_w a_x (X_A - X_B) - S\rho_w (a_z + g)(Z_A - Z_B)$$

$$P_A - 20{,}000 = -0.68\times 1000\,(4.33)(0 - 1.2\cos 30^\circ)$$
$$- 0.68\times 1000\,(2.5 + 9.81)(0 - 1.2\sin 30^\circ)$$

$$\boxed{P_A = 28{,}082.4\ \mathrm{Pa}}$$

This is the pressure at the plug.

* Force $= P \times$ Area

$$= 28{,}082.4 \times \frac{\pi \times 0.04^2}{4}$$

$$\boxed{\text{Force} = 35.3\ \mathrm{N}}$$

This is the force acting on the plug.