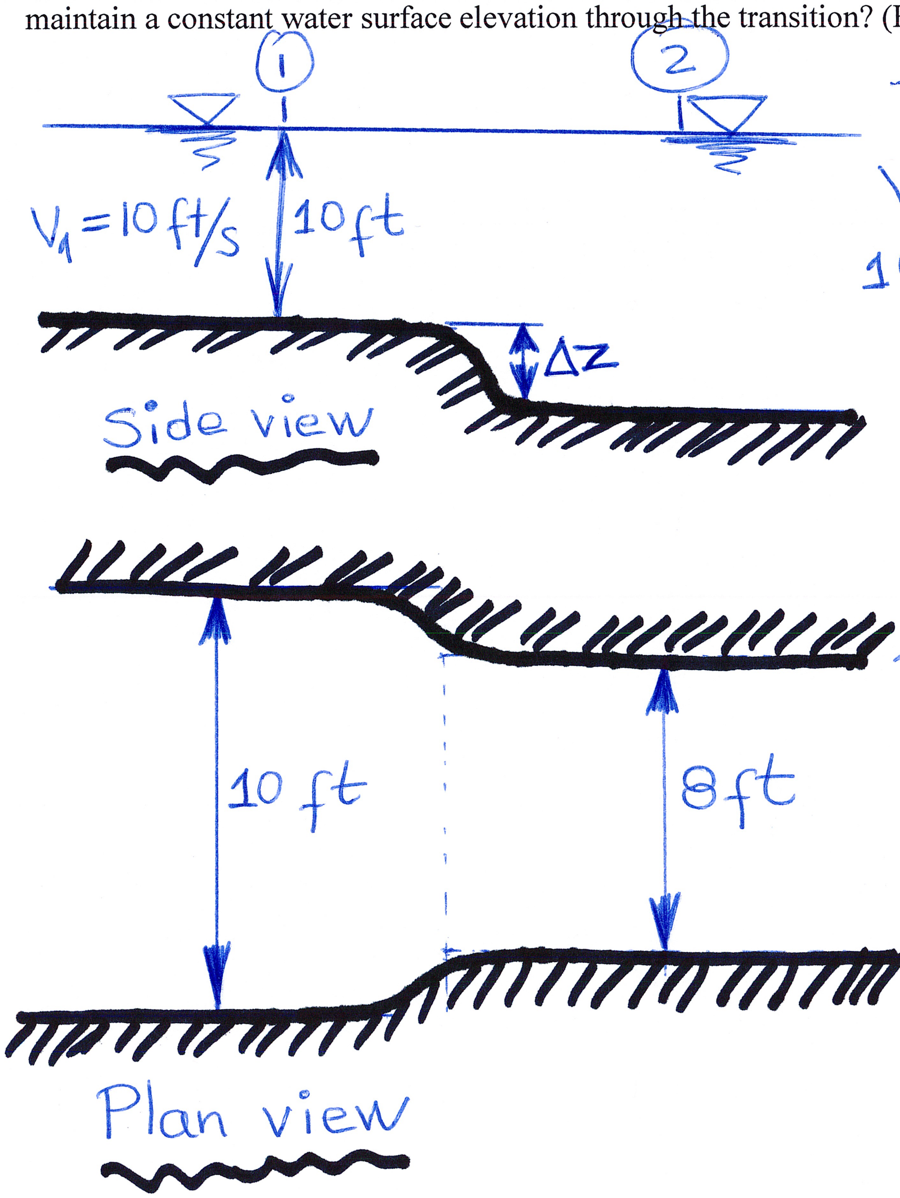
Florida International University
CWR 3201 Fluid Mechanics, Fall 2018
Instructor: Arturo S. Leon, Ph.D., P.E., D.WRE
TA: Thao Do, CEE Undergraduate

Final Exam

Student Name and ID: Solution/Arturo Leon **Date:** ______

- ✓ **You will have two hours to complete the exam. The exam is closed book and closed notes**
- ✓ **The procedure will be graded. Please justify your answers**

1. (15 points) A 10-ft wide rectangular channel is flowing at a depth of 10-ft with a velocity of 10 ft/s. If the channel has a smooth contraction in width from 10 ft to 8 ft, how much should the channel bottom drop to maintain a constant water surface elevation through the transition? (Head loss coefficient = 0)



* Continuity

$$V_1 b_1 Y_1 = V_2 b_2 Y_2$$

$$10(10)(10) = V_2(8)(10+\Delta z)$$

$$125 = V_2(10+\Delta z) \quad \ldots (1)$$

* $E_1 = E_2$

$$\cancel{10+\Delta z} + \frac{V_1^2}{2g} = \cancel{10+\Delta z} + \frac{V_2^2}{2g}$$

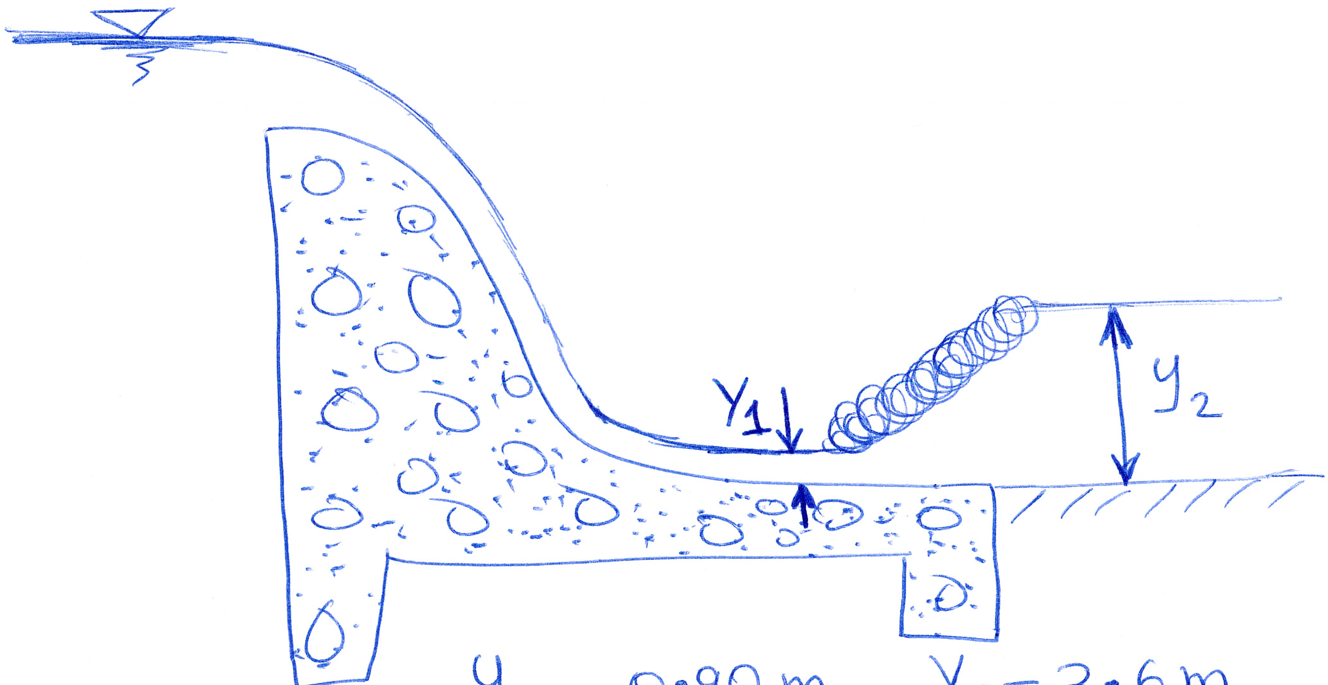
$$V_1 = V_2$$

In (1)

$$125 = 10(10+\Delta z)$$

$$\boxed{\Delta z = 2.50 \text{ ft}}$$

2. (15 points) A hydraulic jump at the base of a spillway of a dam is such that the depths upstream and downstream of the jump are 0.90 and 3.6 m, respectively. If the spillway is 10 m wide, what is the flowrate over the spillway?



$y_1 = 0.90\text{ m}, \quad y_2 = 3.6\text{ m}$

$b = 10\text{ m}$

$Q = ??$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8Fr_1^2} - 1\right]$$

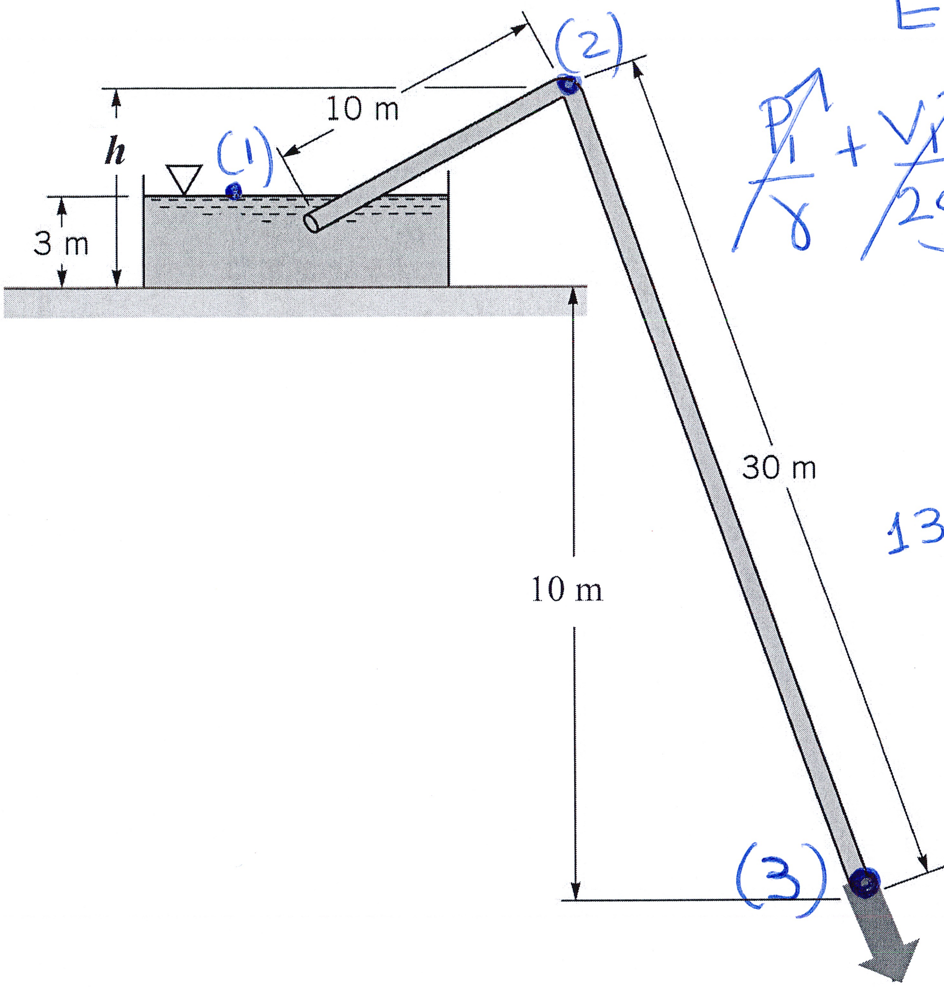
$$\frac{3.6}{0.9} = \frac{1}{2}\left[\sqrt{1+8Fr_1^2} - 1\right] \rightarrow \boxed{Fr_1 = 3.16}$$

$$Fr_1 = \frac{V_1}{\sqrt{9.81 \times 0.9}} \rightarrow \boxed{V_1 = 9.39\text{ m/s}}$$

$$\therefore Q = V_1 \times (0.9)(10)$$

$$\boxed{Q = 84.5\text{ m}^3/\text{s}}$$

3. (20 points) A 40-m long, 12-mm diameter pipe with a friction factor of 0.020 is used to siphon 30°C water from a tank as shown below. Determine the maximum value of $\boldsymbol{h}$ allowed if there is to be no cavitation within the hose. Neglect minor losses. Use P_{vapor} (30°C)= 4.24 kPa, P_{atm} = 101.3 kPa, γ(30°C) = 9.768 kN/m³.



$$E_1 = E_3$$

$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{P_3}{\gamma}} + \frac{V_3^2}{2g} + z_3 + \frac{fL}{D}\frac{V_3^2}{2g}$$

$$13 = \frac{V_3^2}{2\times 9.8}\left(1 + \frac{0.02\times 40}{0.012}\right)$$

$$\boxed{V_3 = 1.94 \text{ m/s}}$$

$V_2 = V_3$ (continuity)

* $E_1 = E_2$

Right before cavitation $P_2 = P_v = 4.24 \frac{kN}{m^2}$

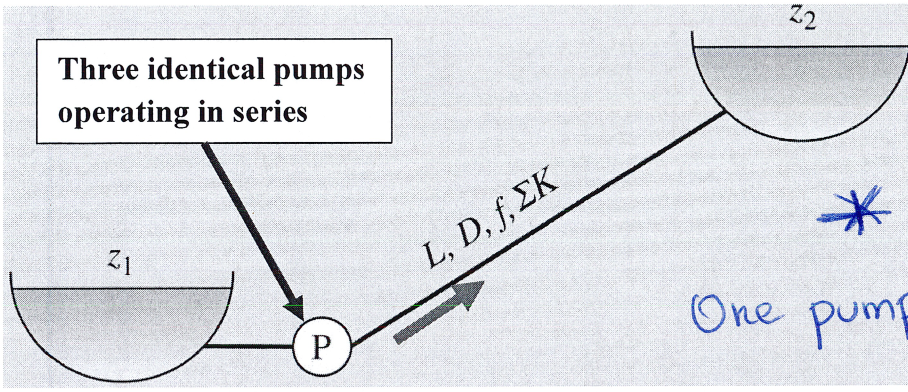
$$\frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}}^{\,0} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \frac{0.02\times 10}{0.012}\left(\frac{V_2^2}{2g}\right)$$

$$\frac{101.3 \text{ kN/m}^2}{9.768 \text{ kN/m}^3} + 3 = \frac{4.24 \text{ kN/m}^2}{9.768 \text{ kN/m}^3} + \underbrace{\frac{1.94^2}{19.6}\left(1 + \frac{0.02\times 10}{0.012}\right)} + h$$

$$13.371 = 0.434 + 3.392 + h$$

$$\therefore \boxed{h = 9.54 \text{ m}}$$

4. (20 points) Water is pumped between two reservoirs in a pipeline with the following characteristics: $D = 300$ mm, $L = 50$ m, $f = 0.025$, $\Sigma K = 4.0$. The radial-flow pump characteristic curve is approximated by the formula $H_P = 22.9 + 10.7Q - 111Q^2$
where H_P is in meters and Q is in m^3/s. Determine the discharge Q_D and pump head H_D if $z_2 - z_1 = 40$ m with **three identical pumps operating in series.**



$Z_2 - Z_1 = 40$ m

* Pump curve:

One pump: $Hp = 22.9 + 10.7Q - 111Q^2$

Three pumps in series:

$Hp = 3(22.9 + 10.7Q - 111Q^2)$

$\boxed{Hp = 68.7 + 32.1Q - 333Q^2}$ ... 1

* System curve:

$Hp = Z_2 - Z_1 + \left(f\frac{L}{D} + \Sigma K\right)\frac{Q^2}{2gA^2}$

$Hp = 40 + \left(0.025 \times \frac{50}{0.3} + 4\right)\frac{Q^2}{2 \times 9.8\left[\pi \times 0.15^2\right]^2}$

$\boxed{Hp = 40 + 83.4Q^2}$ ... 2

1 = 2

$68.7 + 32.1Q - 333Q^2 = 40 + 83.4Q^2$

$28.7 + 32.1Q - 416.4Q^2 = 0$

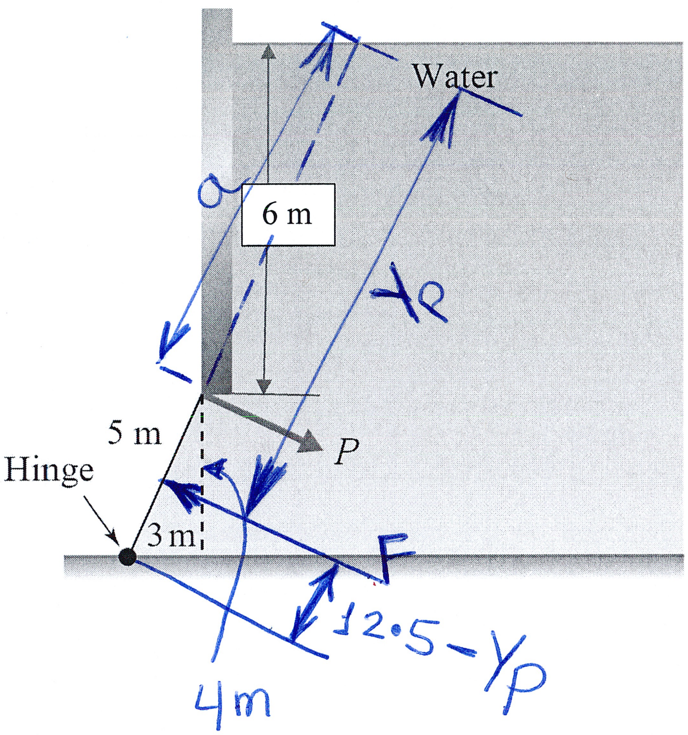
$$Q_1 = -0.22$$

$$Q_2 = 0.304$$

$$\therefore \boxed{Q_D = 0.304\ m^3/s}$$

In (2) $\underline{H_p = 47.71\ m}$

5. (15 points) Calculate the force ***P*** necessary to hold the 4-m wide gate in the position shown in the Figure below.



$$\frac{4}{5} = \frac{6}{a} \rightarrow a = 7.5$$

$$\sum M = 0$$

$$P(5) = F(12.5 - y_p) \quad \ldots \text{①}$$

$$F = \gamma \bar{h} \cdot A = 9810 * 8 \times 5 \times 4$$

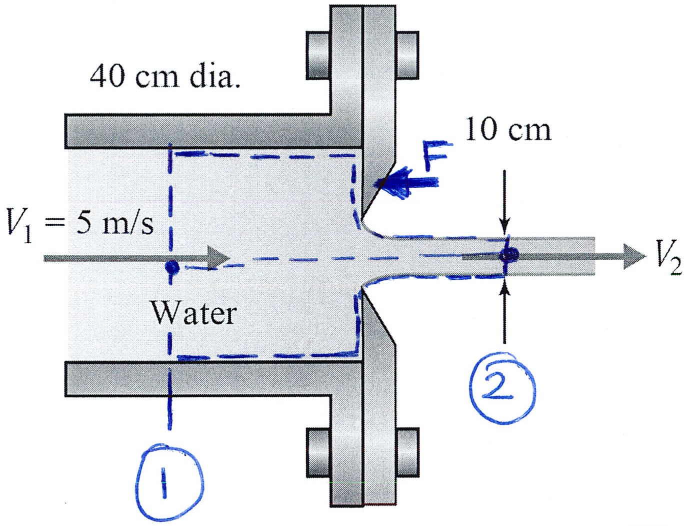
$$\boxed{F = 1570 \text{ KN}}$$

$$* \; y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}} = 10 + \frac{\cancel{4} \times 5^3}{12(5 \times \cancel{4} \times 10)} = 10.208$$

$$\therefore P(5) = 1570 \text{ KN}(12.5 - 10.208)$$

$$\boxed{P = 720 \text{ KN}}$$

6. (15 points) What is the force needed to hold the orifice plate shown below onto the pipe?



Continuity

$$5\left(\frac{\cancel{\pi} \times 0.40^2}{\cancel{4}}\right) = V_2 \frac{\cancel{\pi} \times 0.10^2}{\cancel{4}}$$

$$\boxed{V_2 = 80 \text{ m/s}}$$

Bernoully

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{Z_1} = \cancel{\frac{P_2}{\gamma}}^{0} + \frac{V_2^2}{2g} + \cancel{Z_2}$$

$$\frac{P_1}{\gamma} = \frac{80^2 - 5^2}{2 \times 9.81} \rightarrow P_1 = 3.19 \times 10^6 \text{ Pa.}$$

γ = 9.81 x 1000

Momentum

$$P_1 A_1 - F - \cancel{P_2}^{0} A_2 = \dot{m}\left(V_{2x} - V_{1x}\right)$$

$$3.19 \times 10^6 \; \frac{\pi \times 0.4^2}{4} - F = 1000 \times 5 \times \frac{\pi \times 0.4^2}{4}(80 - 5)$$

$$F = 353{,}000 \text{ N}$$

$$\boxed{F = 353 \text{ kN}}$$