## Water Surface Profiles

## Computation Procedure

1. Data Collection: Obtain all channel and flow information including channel shape (or $x$-sections at pertinent locations), channel slope, and Manning roughness factor.
2. Control Section: Determine the location of the control section (boundary condition) including depth and flow.
3. Channel Depths: Compute normal and critical depths .
4. Channel Classification: Determine if the channel is mild $(M)$, steep (S), critical (C), horizontal (H), or adverse (A).
5. Water Surface Profile Classification: Classify the flow as Type 1, 2, or 3 based on flow depth ( $y$ ), $y_{n}$ and $y_{c}$.

## Standard Step Method Review

(Gradually Varied Flow)
Computational Algorithms: Energy Balance \& Manning eq'n:

$$
z_{2}+y_{2}+\left(V_{2}\right)^{2} / 2 g=z_{1}+y_{1}+\left(V_{1}\right)^{2} / 2 g+\Delta L\left(S_{e}\right)_{\text {avg }}
$$

For SI: $S_{e}=n^{2} V^{2} /\left(R_{h}\right)^{4 / 3}$ For BG: $S_{e}=n^{2} V^{2} /\left[2.22\left(R_{h}\right)^{4 / 3}\right]$
$\left(S_{e}\right)_{\text {avg }}$ is the average EGL at upstream \& downstream sections.


Process: Use successive computations starting at the control section. Find flow depths by balancing energy between adjacent $x$-sections going upstream for subcritical flow.

## Water Surface Profile Computations <br> (Example Problem - Standard Step Method)

Given: Concrete channel with $S_{0}=0.001$, that ends in a waterfall (drop-off). $\rightarrow$ Find: Depths of flow at $5 \mathrm{~m}, 25 \mathrm{~m}$, and 100 m upstream of the waterfall.
Solution Steps: (from previous slides)


1. Data Collection: Available from problem statement.
2. Control Section: Critical depth $\left(y_{c}\right)$ occurs at drop-off.
3. Channel Depths: Determine $y_{n}$ and $y_{c}$ for the channel.

$$
q=Q / b=10 \mathrm{cms} / \mathrm{m} ; y_{c}=
$$

## Water Surface Profile Computations <br> (Example Problem - Standard Step Method)

3b. Channel Depths: Determine $y_{n}$.
From Table 6.1: $A=10 y \& P=10+2 y$ $Q=(1 / n) A R_{h}{ }^{2 / 3} S_{0}{ }^{1 / 2}=(1 / n)\left(A^{5 / 3} / P^{2 / 3}\right) S_{0}^{1 / 2}$ Thus, $Q n / S_{0}^{1 / 2}=\left(100^{*} 0.013\right) /(0.001)^{1 / 2}=$ $41.1=\left(10 y_{n}\right)^{5 / 3} /\left[10+2 y_{n}\right]^{2 / 3} ; y_{n}=2.79 m$

4. Channel Classification: Since $y_{n}>y_{c} \rightarrow$ Channel is $\qquad$
5. Water Surface Profile Classification: Depth rises from $y_{c}$ at the drop-off to $y_{n}$ upstream $\rightarrow$ M2. Sketch it.


## Water Surface Profile Computations



In subcritical flow, calculations progress upstream starting at the control section ( $y_{1}=y_{c}=2.17 \mathrm{~m}$; boundary condition). Try to balance energy between sections 1 and 2 with a trial depth $\left(y_{2}=2.35 \mathrm{~m}\right)$. Assume $z_{1}=0.00 \mathrm{~m}$. A table is helpful.

| Sec. | $\mathbf{U} / \mathbf{D}$ | $\mathbf{y}$ | $\mathbf{z}$ | $\mathbf{A}$ | $\mathbf{V}$ | $\mathbf{V}^{\mathbf{2}} / \mathbf{2 g}$ | $\mathbf{P}$ | $\mathbf{R}_{\mathbf{h}}$ | $\mathbf{S}_{\mathbf{e}}$ | $\mathbf{S}_{\mathbf{e ( a v g})}$ | $\mathbf{h}_{\mathbf{L}}$ | Total E |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\#$ |  | $(\mathrm{~m})$ | $(\mathrm{m})$ | $\left(\mathrm{m}^{2}\right)$ | $(\mathrm{m} / \mathrm{sec})$ | $(\mathrm{m})$ | $(\mathrm{m})$ | $(\mathrm{m})$ |  |  | $(\mathrm{m})$ | $(\mathrm{m})$ |
| 1 | D | 2.17 | 0.000 | 21.70 | 4.608 | 1.0824 | 14.340 | 1.513 | 0.002066 | 0.001851 | 0.0093 | 3.262 |
| 2 | U | 2.35 | 0.005 | 23.50 | 4.255 | 0.9229 | 14.700 | 1.599 | 0.001637 | $\Delta \mathrm{~L}=$ | 5 | 3.278 |

Note: The trial depth of 2.35 m is incorrect since energies don't balance Try a smaller depth at Section 2.

$$
z_{u}+y_{u}+\left(V_{u}\right)^{2} / 2 g=z_{D}+y_{D}+\left(V_{D}\right)^{2} / 2 g+\Delta L\left(S_{e}\right)_{\text {avg }}
$$

For SI: $S_{e}=n^{2} V^{2} /\left(R_{h}\right)^{4 / 3}$ For BG: $S_{e}=n^{2} V^{2} /\left[2.22\left(R_{h}\right)^{4 / 3}\right]$

## Water Surface Profile Computations

Try to balance energy between sections 1 and 2 again with a new trial depth $\left(y_{2}=2.25 \mathrm{~m}\right)$. Fill in the blanks in the table.

| Sec. | $\mathbf{U} / \mathbf{D}$ | $\mathbf{y}$ | $\mathbf{z}$ | $\mathbf{A}$ | $\mathbf{V}$ | $\mathbf{V}^{2} / \mathbf{2 g}$ | $\mathbf{P}$ | $\mathbf{R}_{\mathbf{h}}$ | $\mathbf{S}_{\mathbf{e}}$ | $\mathbf{S}_{\mathbf{e}(\mathbf{a v g})}$ | $\mathbf{h}_{\mathbf{L}}$ | Total E |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\#$ |  | $(\mathrm{~m})$ | $(\mathrm{m})$ | $\left(\mathrm{m}^{2}\right)$ | $(\mathrm{m} / \mathrm{sec})$ | $(\mathrm{m})$ | $(\mathrm{m})$ | $(\mathrm{m})$ |  |  | $(\mathrm{m})$ | $(\mathrm{m})$ |
| 1 | D | 2.17 | 0.000 | 21.70 | 4.608 | 1.0824 | 14.340 | 1.513 | 0.002066 | 0.001962 | 0.0098 | 3.262 |
| 2 | U | 2.25 | 0.005 |  |  | 1.0068 |  |  |  | $\Delta \mathrm{~L}=$ | 5 |  |
| Note: The trial depth of 2.25 m is correct. Now balance energy between sections \#2 and \#3. |  |  |  |  |  |  |  |  |  |  |  |  |

$$
z_{u}+y_{u}+\left(V_{u}\right)^{2} / 2 g=z_{D}+y_{D}+\left(V_{D}\right)^{2} / 2 g+\Delta L\left(S_{e}\right)_{\text {avg }}
$$

For SI: $S_{e}=n^{2} V^{2} /\left(R_{h}\right)^{4 / 3}$ For BG: $S_{e}=n^{2} V^{2} /\left[2.22\left(R_{h}\right)^{4 / 3}\right]$
Note: Errors propagate, so use more significant figures than normal.

## Water Surface Profile Computations

Now balance energy between sections $2\left(y_{2}=2.25 \mathrm{~m}\right)$ and 3 (trial depth $\rightarrow y_{3}=2.35 \mathrm{~m}$ ). Fill in the blanks in the table.

| Sec. | U/D | y | z | A | V | $\mathrm{V}^{2} / 2 \mathrm{~g}$ | P | $\mathrm{R}_{\mathrm{h}}$ | Se | $\mathbf{S e}_{\text {e(avg) }}$ | hL | Total E |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \# |  | (m) | (m) | $\left(\mathrm{m}^{2}\right)$ | (m/sec) | (m) | (m) | (m) |  |  | (m) | (m) |
| 2 | D | 2.25 | 0.005 | 22.50 | 4.444 | 1.0068 | 14.500 | 1.552 | 0.001858 | 0.001748 | 0.0350 |  |
| 3 | U | 2.35 |  | 23.50 | 4.255 | 0.9229 |  |  |  | $\Delta \mathrm{L}=$ |  |  |
|  | Note: The trial depth of 2.35 m is correct. Now balance energy between sections \#3 and \#4. |  |  |  |  |  |  |  |  |  |  |  |

$$
z_{u}+y_{u}+\left(V_{u}\right)^{2} / 2 g=z_{D}+y_{D}+\left(V_{D}\right)^{2} / 2 g+\Delta L\left(S_{e}\right)_{\text {avg }}
$$

For SI: $S_{e}=n^{2} V^{2} /\left(R_{h}\right)^{4 / 3}$ For BG: $S_{e}=n^{2} V^{2} /\left[2.22\left(R_{h}\right)^{4 / 3}\right]$
Note: Energy balance should be very close before proceeding.

## Water Surface Profile Computations (Standard Step Method - Final Comments)



Notes: X-section spacing should be quite close when depths change rapidly.

| Section \# | Depth (y) | Distance* |
| :--- | :--- | :--- |

1
2.17 m

0 5

25 100
*Distance upstream from waterfall. Normal depth ( 2.79 m ) has not been reached in the channel. It will be approached asymptotically as additional sections are added further upstream. An iterative procedure is not necessary in the direct step method (see textbook).
However, it will only work with prismatic channels. Also, it is easy to program a spreadsheet to do the calculations.

## Hydraulic Design of Open Channels

## (Generally Designed for Uniform Flow)

Design Requirements: Channel size, shape, slope, alignment, and liner.
Liner Types:
Primary Design Equation: $\qquad$
Design Constraints: Topography, right-of-way, adjacent structures, proposed liner, depth of groundwater table, cost, $N_{F} \ll 1.0$ (subcritical), and freeboard.
Slope Stability: Based on channel liner material (Table 6.6)


Hydraulic Design of Open Channels (Unlined and Rigid Boundary Channels)

Unlined Channels: excavated in existing soil on site Design channel so that the average $V$ doesn't exceed $V_{\text {max }}$ for channel material Design channel so that the shear stress ( $\tau$ ) on the channel liner doesn't exceed the $\tau_{\text {allowable }}$ Rigid (non-erodible) Boundary Channels: Use the "best hydraulic" section concept (Section 6.3). The conveyance capacity of a channel section for a given flow area is maximized when the wetted perimeter is minimized.

Homework:

