PROBLEM 2.31

KNOWN: Air within a piston-cylinder assembly undergoes two processes in series.

FIND: Determine the total work.

SCHEMATIC & GIVEN DATA:

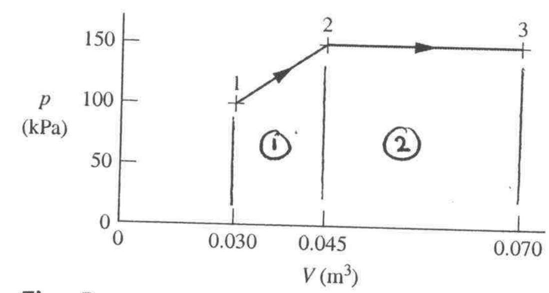


Fig. P2.32

ENGR. MODEL:

1. The air within the piston-cylinder assembly is the closed system.
2. The two-step p-V relation during expansion is specified.

ANALYSIS: Since volume change is the work mode, Eq. 2.17 applies. Furthermore the integral can be evaluated geometrically in terms of the total area under the process line:

$$W = \int_{V_1}^{V_2} p\,dV = \overset{①}{p_{ave}(V_2 - V_1)} + \overset{②}{p_2[V_3 - V_2]} = \left(\frac{p_2 + p_1}{2}\right)(V_2 - V_1) + p_2(V_3 - V_2):$$

$$= \left\{\left(\frac{150+100}{2}\right)(kPa)[0.045 - 0.030]\,m^3 + (150\,kPa)(0.070 - 0.045)\,m^3\right\}\left|\frac{10^3\,N/m^2}{1\,kPa}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

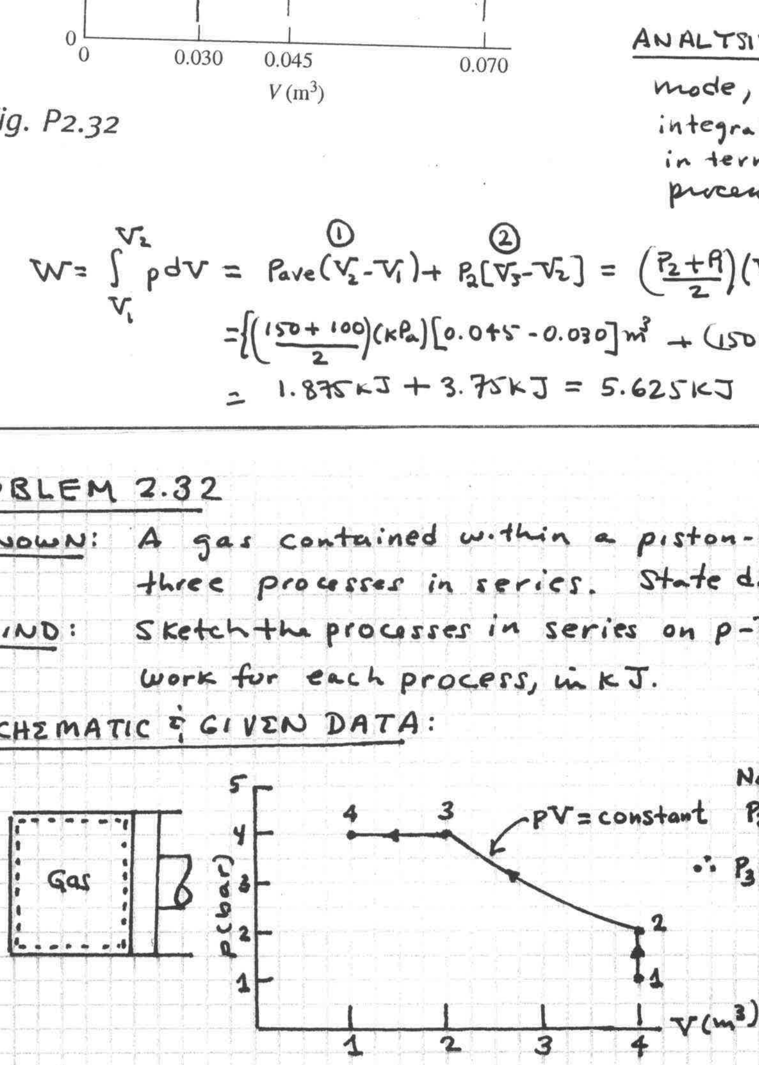
$$= 1.875\,kJ + 3.75\,kJ = 5.625\,kJ \quad \longleftarrow$$

PROBLEM 2.32

KNOWN: A gas contained within a piston-cylinder assembly undergoes three processes in series. State data are provided.

FIND: Sketch the processes in series on p-V coordinates and evaluate the work for each process, in kJ.

SCHEMATIC & GIVEN DATA:



Note: $p_2 V_2 = p_3 V_3$

$\therefore\ p_3 = p_2\left[\frac{V_2}{V_3}\right]$

$= 2\,bar\left[\frac{4}{2}\right]$

$= 4\,bar$

ENGINEERING MODEL:

1. The gas within the piston-cylinder is the closed system.
2. The gas experiences three processes, in series, as shown in the sketch.

ANALYSIS: The work is given by Eq. 2.17: $W = \int p\,dV$

Process 1-2: V is constant. Thus, the piston does not move, and $W_{12} = 0$. ⟵

Process 2-3: $W_{23} = \int_2^3 \frac{C}{V}\,dV = C \ln\frac{V_3}{V_2} = p_2 V_2 \ln\frac{V_3}{V_2}$

$$= \left(2\times10^5\,\frac{N}{m^2}\right)(4\,m^3)\left|\frac{1\,kJ}{10^3\,N\cdot m}\right| \ln\left[\frac{2}{4}\right] = -554.5\,kJ \quad \longleftarrow$$

Process 3-4: $W_{34} = p[V_4 - V_3]$

$$= \left(4\times10^5\,\frac{N}{m^2}\right)(1-2)\,m^3\left|\frac{1\,kJ}{10^3\,N\cdot m}\right| = -400\,kJ \quad \longleftarrow$$