P2.1.19.

A digital communication system sends information in 8-bit words. No check bit is provided, and the probability of an error on a given bit is constant, regardless of position in the word. On one occasion when conditions were poor, 10 out of 100 words transmitted were detected incorrectly. Estimate the probability that a given word contained exactly two errors.

A2.1.19. 4.43×10^{-3}

2.1.19 solution. Equating the relative frequency to a probability, we have $P[\text{no errors}] = 0.9 = B_8(0, p)$, where p is the probability of an error. Thus $(1-p)^8 = 0.9$ and $p = 0.01308$. Thus the probability of exactly two errors would be $B_8(2, p) = 4.429\times 10^{-3}$

P2.1.20.

The suitors of a princess bride take part in a high-stakes Easter egg hunt for her hand. Our hero, who truly loves the princess, hires short actors and dresses them as children to represent him in the hunt. He must acquire two golden eggs to get the princess, and each "child" is allowed one golden egg and has a probability of 0.4 of finding such an egg. What is the minimum number of actors that he has to hire to have a probability of at least 0.95 of getting at least two golden eggs?

A2.1.20. 10

2.1.20 solution. The process is binomial: he needs two or more successes with probability exceeding 0.95 and the number of trials, which are the actors, is unknown. This calls for a trial and error solution. Below we give the *Mathematica* code, showing that 10 actors give a probability of 2 or more eggs

```
Needs["Statistics`DiscreteDistributions`"]
n = 10;
p = 0.4
prob = Sum[PDF[BinomialDistribution[n, p], k], {k, 2, n}]

0.4

0.953643
```

P2.1.21.

An engineer drives to work along the same route every day, at a speed much greater than the legal limit. The police radar is set up once every four weeks on work days along that route, but at random times during the time when people are commuting. Assume 240 work days per year. If the police are out, there is a 0.2 chance the engineer will get a ticket. Find the probability that she will get fewer than two tickets in a year.

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Chapter 5

A2.1.22.

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A2.1.23.

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A2.1.24.

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A2.1.21. 0.2749

2.1.21 solution. This works out to $n = 12$ trials, $p = 0.2$, and $k \leq 1$; so $B_{12}(0, 0.2) + B_{12}(1, 0.2) = 0.2749$.

P2.1.22.

An automobile has four tires and one spare. A 3000-mile trip is undertaken. Assume that in each 500-mile stretch, the car has a probability of 0.05 of having one tire failure. Find the probability of completing the trip with no tires repaired along the way. Use a binomial model, but clearly, the model has to be undertaken with caution. We will develop a better model in Chapter 5.

A2.1.22. 0.967

2.1.22 solution. This works out to $n = 6$ trials, $p = 0.05$, and $k \leq 1$; so $B_6(0, 0.05) + B_6(1, 0.05) = 0.9672$.

P2.1.23.

A telemarketer gets paid \$1 for each sale he makes. He can make six calls/hour, and each call has a probability of 0.6 of making a sale.
a. Find the probability that he makes at least \$18 in working four 1-hour sessions.
b. Find the probability that he makes at least three sales in each of the four 1-hour periods he works.

A2.1.23. **(a)** 0.960; **(b)** 0.454

2.1.23 solution. (a) $n = 24$ and $k \geq 18$, so $\sum_{k=18}^{24} B_{24}(k, 0.6) = 0.960$. (b) This is $n = 4$ and $p = P[k \geq 3] = 0.8208$ and we require success in all four periods, $B_4(4, 0.8208) = 0.8208^4 = 0.4539$.

P2.1.24.

An experienced fly fisherman gets a fish to strike his lure about 20% of the casts he makes, but an inexperienced fisherman gets a strike on only 10% of his casts. An experienced and an inexperienced fisherman go fishing together. What is the probability that the experienced fisherman has a strike before the inexperienced, considering the strikes independent events? Consider that they fish a long time and cast simultaneously.

A2.1.24. 0.643

2.1.24 solution. The sample space is an infinite array, and we are summing the probabilities off the diagonal on the side of the inexperienced fisherman. A partition would be that the experienced fisherman strikes on the 1st, 2nd, ... cast. Letting n = the cast on which the experienced fisherman makes his first strike and m = the cast on which the inexperienced fisherman makes his first strike, we can express the answer as $\sum_{n=1}^{\infty} P[m > n \mid n]\, P_1(n, 0.2)$. The conditional probability is itself an infinite sum, $P[m > n \mid n] = \sum_{m=n+1}^{\infty} P_1(m, 0.1)$. Both these sums can be performed in closed form. The second has the value 0.9^n, and the first the value $\frac{0.2\times0.9}{1-0.8\times0.9} = 0.6429$.

P2.1.25.

A duck hunter has a limit of three ducks. She has a probability of 0.6 hitting a duck. How many shells should she take to have a probability of 0.9 or greater of getting her limit and not running out of shells?

A2.1.25. seven shells gives 0.9037

2.1.25 solution. This requires $B_n(\geq 3, 0.6] \geq 0.9$. Since n is the unknown, we have to use trial and error. For $n = 6$, we have $1 - B_6(0, 0.6) - B_6(1, 0.6) - B_6(2, 0.6) = 0.8208$. For $n = 7$ we get 0.9037 so that's the answer.

P2.1.26.

A medical practice has 50 appointment slots per day. Experience has shown that 20% of the appointments will not show up. Of these broken appointments, 75% will call and cancel, and 25% will simply be no-shows.

a. How many can be expected to phone in to cancel during a typical day?

b. If on a given day 12 people fail to keep their appointments, what is the probability that 10 or more called to cancel?

c. On a day when the first 10 appointments show up, what is the probability that more than 2 appointments are not kept?

A2.1.26. **(a)** ≈7.5; **(b)** 0.3907; **(c)** 0.9921

2.1.26 solution. (a) $P[\text{phone in} \mid \text{no show}] \times P[\text{no show}] = 0.75\times0.2 = 0.15$. The number phoning in to cancel will be about $50\times0.15 = 7.5$. (b) $B_{12}(\geq 10,\ 0.75) = 0.3907$. (c) $P[K > 2 \mid \text{first 10 show up}] = 1 - B_{40}(0, 0.2) - B_{40}(1, 0.2) - B_{40}(2, 0.2) = 0.9921$.

P2.1.27.

A door-to-door salesman gets to give his presentation on one out of every three doors he knocks on. If he gives the presentation, he makes a sale one time out of two. It takes 1 hour to make the presentation, regardless of whether a sale is made.

a. What is the probability that he will make only one presentation out of the first five homes he visits?

b. The salesman must make two sales simply to cover his daily expenses. After that he makes profit to live on, buy food and other necessities, and so on. What is the probability that he will cover his expenses after he has spent the first 4 hours selling? Neglect knocking and walking time.

A2.1.27. **(a)** 0.329; **(b)** 0.6875

2.1.27 solution. (a) $B_5(1, \frac{1}{3}) = 0.3292$. (b) $B_4(\geq 2, \frac{1}{2}) = 0.6875$

P2.1.28.

The corporate jet of FatCats, Inc., offers steak or chicken for the executive traveling in style. Experience has shown that 60% of the passengers prefer steak. If there are nine passengers, how may steaks should the jet carry to have at least a 90% probability of giving a passenger a steak if he or she requests it. (Ignore chicken orders and vegetarians.)

A2.1.28. 7 gives P[not run out] = 0.929

2.1.28 solution. Find k such that $P[\le k \text{ of } 9] \ge 0.9$. Best done as $1 - \sum_{i=k+1}^{9} B_9(i, 0.6) \le 0.9$. So we start with $k = 9$ and work them out subtracting until it falls under 0.9. For $k = 8$, the result is 0.9899. For $k = 7$, the result is 0.9295, and for $k = 6$ the result is 0.8268. So they have to carry 7.

P2.1.29.

It is said that 90% of small-company start-ups fail in 3 years or less. Assume the failures are described as binomial trials on a yearly basis.

a. An engineer started a company in 2004 What is the probability it will be operating in 2006 (2 years)?

b. If the company lasts 2 years, what is the probability it will last 3 more years?

A2.1.29. **(a)** 0.215; **(b)** 0.1

2.1.29 solution. (a) This is geometric. $P[\text{survive the year}]^3 = 0.1$, so $P[\text{survive}] = 0.464$. Therefore, $P[\text{survive two years}] = 0.464^2 = 0.215$. (b) By the assumption of binomial trials, trials are independent, so the past doesn't matter. The answer is 0.1.

P2.1.30.

If Sam studies, ST, for a test, the probability of getting a question correct is 0.8. If Sam does not study for the test, $\overline{ST}$, the probability that Sam will get the question correct is 0.6. There are 10 questions on the test, and we assume the conditions for independent trials. There is a 70% chance that Sam will study and a 30% chance that he will not study.

a. If Sam studies, find the probability that he makes 70 or more on the test.

b. What is the probability that Sam makes 70 or more on the test?

c. Given that Sam made 70 or more on the test, what is the probability that he studied?

A2.1.30. **(a)** 0.8791; **(b)** 0.7301; **(c)** 0.8429

2.1.30 solution. This is a binomial model.

(a) $P[\ge 70 \mid \text{ST}] = B_{10}(7, 0.8) + B_{10}(8, 0.8) + B_{10}(9, 0.8) + B_{10}(10, 0.8) = 0.8791$. (b) Same calculation with $p = 0.6$, which gives 0.3823 and then the law of total probability: $P[\ge 70 \mid \text{ST}] = 0.8791 \times 0.7 + 0.3823 \times 0.3 = 0.7301$. (c) $P[\text{ST} \mid \ge 70] = \frac{0.8791 \times 0.7}{0.7301} = 0.8429$

P2.1.31.

I went to graduation last Saturday. The faculty's role was to walk in at the beginning, stand up near the end, and walk out at the end, wearing colorful attire. As I was reading the names of

2.4.3 The Definition and Algebra of Expectation   Back

Exercises on Expectation

P2.3.1.

A container contains balls with numbers, 1, 2, 3,..., m written on them. There is one ball with 1 on it, two balls with 2 , ..., up to m balls with m written on them. The balls are drawn one at a time, with replacement, until a 4 is drawn. On average it takes seven draws to get a 4. What is m?

A2.3.1. 7

2.3.1 solution. $1 + 2 + ... + m = \frac{m}{2}(m+1)$, so $P[4] = \frac{4\times 2}{m(m+1)}$. The process of selection is geometric so $E[N] = \frac{1}{p} = \frac{m(m+1)}{8} = 7 \Rightarrow m = 7$.

P2.3.2.

A random variable takes on five values. The PMF is

$P_X(1) = 0.1$

$P_X(2.1) = 0.2$

$P_X(2.6) = 0.2$

$P_X(3) = 0.15$

$P_X(3.7) = 0.35$

a. Show that the PMF is normalized.

b. Find the mean of X.

c. Find the variance of X.

d. Find the probability that on a given performance of the experiment X falls within one standard deviation of the mean.

A2.3.2. **(a)** The probabilities add to 1.0; **(b)** 2.79; **(c)** 0.7193; **(d)** 0.55

2.3.2 solution. (a) The values add to 1. (b) $\mu_X = 1\times 0.1 + 2.1\times 0.2 + 2.6\times 0.2 + 3\times 0.15 + 3.7\times 0.35 = 2.79$. (c) The mean square value is computed similarly to be $E[X^2] = 8.48$, so $\sigma_X^2 = 8.48 - 2.79^2 = 0.7193$. (d) $\sigma_X = \sqrt{0.719} = 0.848$. $P[2.79 - 0.848 < X \le 2.79 + 0.848] = 0.2 + 0.2 + 0.15 = 0.55$

P2.3.3.

A integer is chosen at random in the range $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. A random variable is defined on the outcomes $X = \frac{1}{i}$, where i is the outcome of the chance experiment.

a. Give the PMF of X.

b. Determine the mean of X.

	Q	D	N
T	1	0	0
S	1	1	1
F	2	0	0

The problem asks for the marginal PMFs for Q and D. Q takes on values 1 and 2, with two of the outcomes favoring 1 and one outcome favoring 2: $P_Q(q) = \frac{2}{3}$ for $q = 1$, $= \frac{1}{3}$ for $q = 2$, zow. D takes on values 0 and 1, with two outcomes favoring 0 and one favoring 1: $P_D(d) = \frac{2}{3}$ for $d = 0$, $= \frac{1}{3}$ for $d = 1$, zow.

P2.2.13.

A student throws darts. He has a 90% chance of hitting the target. If he hits the target, he has a 20% chance of a bull's-eye (10 points), a 30% chance of hitting the second ring (5 points), and a 50% chance of hitting the outer ring (2 points). Zero points are awarded when the target is missed. Let X = the points awarded on a throw. Determine and sketch the PMF, $P_X(x)$.

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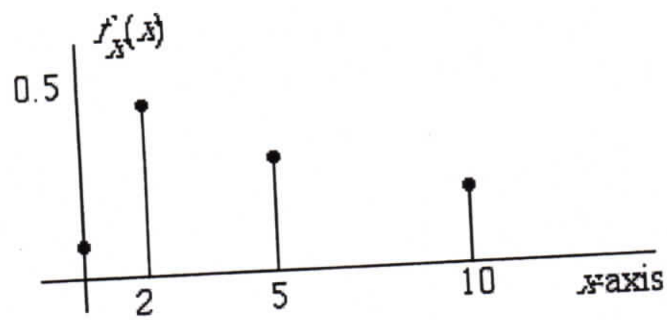
A2.2.13. $P_X(x) = 0.18, 0.27, 0.45, 0.1$, for $x = 10, 5, 2$, and 0, respectively, zow



2.2.13 solution. Here the chance experiment has four outcomes, which are hitting the bullseye, hitting the second ring, hitting the outer ring, and missing the target altogether. The probabilities are given in terms of the partition, H and $\overline{H}$, where H stands for hitting the target, and $P[H] = 0.9$. Letting B, S, and T standing for hitting the bullseye, second, and third rings, respectively, we have, for example, $P[B] = P[B \mid H]\,P[H] = 0.20\times0.90 = 0.18$. Similarly, $P[S] = 0.30\times0.9 = 0.27$, and $P[T] = 0.50\times0.90 = 0.45$. The values of $X(s)$, the random variable, are clearly defined in the problem statement: Bullseye = 10 points, and so forth. The probability that $X = 10$ is the probability that a bullseye is hit, which is 0.18. Thus $P_X(10) = P[X = 10] = 0.18$. The same reasoning applies for the other values of X have probability associated with them, namely 0, 2, and 5. Note that $\sum_{\text{all } x_i} P_X[x] = 1$.

P2.2.14.

A man takes his wife to dinner in a restaurant where meals cost \$6, \$8, or \$10. His wife has good taste and is more likely to pick the more expensive dishes. Let $\frac{6}{24}$, $\frac{8}{24}$, and $\frac{10}{24}$ be the probabilities of her choosing the \$6, \$8, or \$10 meals, respectively. He, being from a small